Analysis of the Effect of Flow Rate on the Doppler Continuity Equation for Stenotic Orifice Area Calculations
A Numerical Study

Curt G. DeGroff, MD; Robin Shandas, PhD; Lilliam Valdes-Cruz, MD

Background—Flow-rate dependencies of the Doppler continuity equation are addressed in this study.

Methods and Results—By use of computational fluid dynamic (CFD) software with turbulence modeling, three-dimensional axisymmetric models of round stenotic orifices were created. Flow simulations were run for various orifice area sizes (0.785, 1.13, 1.76, and 3.14 cm²) and flow rates (0.37 to 25.0 L/min). Reynolds numbers ranged from 100 to 8000. Once adequate convergence was obtained with each simulation, the location of the vena contracta was determined. For each run, maximum and average velocities across the cross section of the vena contracta were tabulated and vena contracta cross-sectional area (effective orifice area) determined. The difference between the maximum velocity and the average velocity at the vena contracta was smallest at low-flow states, with more of a difference at high-flow states. At lower-flow states, the velocity vector profile at the vena contracta was parabolic, whereas at high-flow states, the profile became more flattened. Also, the effective orifice area (vena contracta cross-sectional area) varied with flow rate. At moderate-flow states, the effective orifice area reached a minimum and expanded at low- and high-flow states, remaining relatively constant at high-flow states.

Conclusions—We have shown that significant differences exist between the maximum velocity and the average velocity at the vena contracta at low flow rates. A likely explanation for this is that viscous effects cause lower velocities at the edges of the vena contracta at low flow rates, resulting in a parabolic profile. At higher-flow states, inertial forces overcome viscous drag, causing a flatter profile. Effective orifice area itself varies with flow rate as well, with the smallest areas seen at moderate-flow states. These flow-dependent factors lead to flow rate-dependent errors in the Doppler continuity equation. Our results have strong relevance to clinical measurements of stenotic valve areas by use of the Doppler continuity equation under varying cardiac output conditions. (Circulation. 1998;97:1597-1605.)

Key Words: echocardiography • stenosis • blood flow

The impetus for assessing valve areas by Doppler techniques arose from the fact that Doppler-estimated pressure gradients across stenotic valves derived from the simplified Bernoulli equation are a flow-dependent index of stenosis severity.1 Valve area measurement methods using Doppler techniques were developed as an attempt to provide a flow-independent assessment of valve stenosis. Originally, a modified form of the Gorlin and Gorlin equation2 was used in the Doppler quantification of valve areas.3 Later, the continuity equation was introduced4 as an alternative Doppler method to estimate valve areas, and it has since become the most commonly used technique.5-14 Nonetheless, the assumption that the Doppler continuity equation is flow independent has not been proved and may be erroneous. Although some investigators have observed a marked flow dependence of valve areas computed from the Doppler continuity equation in the clinical setting,6,7 others have reported no dependence,8,12 and still others have seen results varying according to whether low or high flow rates are used in in vitro flow models.9 Clinical studies examining the effect of increasing flow volume through exercise or dobutamine infusion on Doppler-calculated valve areas have also been performed.7,9-12 Many of these investigators have found variations in Doppler-calculated valve area as a result of changing flow conditions and conclude that there may be a flow dependence inherent in this method.

The purpose of this investigation was to use numerical modeling experimentation to (1) determine whether the vena contracta cross-sectional area, called the "effective" orifice area, changes with flow rate; (2) determine whether orifice areas derived from the Doppler continuity equation accurately measure effective orifice areas; and (3) determine whether the results of the Doppler continuity equation are flow dependent and if so provide explanations for such
Flow Rate and Doppler Continuity Equation

Figure 1. Streamlines through an orifice. Streamlines are lines drawn in a flow field such that they are always tangent (parallel) to direction of flow. Vena contracta represents a contraction in edges of flow streamlines as they move through an orifice.

Theoretical Considerations

Vena Contracta

Fig 1 shows schematically the flow dynamics around a stenotic orifice. Streamlines are lines drawn within a flow field that are always parallel to the direction of flow. The vena contracta represents a contraction in the edges of the flow streamlines as they move through an orifice. For orifices without a smoothly tapering proximal geometry, inertia prevents proximal streamlines entering from the side from changing direction instantly; in this region, these streamlines are directed almost perpendicular to the general flow direction. As the flow passes through the orifice, the streamlines change direction to run parallel to the main flow direction but not before “squeezing” the main flow and causing a constriction in the cross-sectional area of flow immediately distal to the orifice. The mechanism that causes this reduction in distal flow area has been called the vena contracta effect. The resulting constricted area (cross-sectional area of the vena contracta) is the effective orifice area reflecting the actual area available for flow, which is usually smaller than the true orifice area. Flow separation just distal to the orifice causes a recirculation zone to form. Through the middle of the recirculation zone, the mainstream “fresh” flow continues to accelerate from the orifice to its highest velocity, presumably where the cross-sectional area of the mainstream flow is narrowest (ie, at the vena contracta). Past the vena contracta, blood decelerates again to fill the vessel. Velocity and pressure vary inversely along the centerline of blood flow through the orifice. Proximal to the orifice, pressures are high and velocities are low. Approaching the vena contracta, pressures drop and velocities increase. Past the vena contracta, in what is called the pressure recovery zone, pressures rise toward their original magnitude and velocities decrease.

Doppler Continuity Equation

The Doppler continuity equation is derived from the concepts of conservation of mass (continuity equation) and the control volume theory, whereby flow rate through a reference point \(Q_{\text{reference}}\) and through an orifice along a flow stream must be equal. We assume there is a means to accurately measure flow rate through a reference location. The Doppler continuity equation method assumes a uniform velocity profile along the cross section of the orifice \(V_{\text{Doppler}}\). The Doppler continuity equation orifice area \(OA_{\text{DCE}}\) is given as:

\[
OA_{\text{DCE}} = Q_{\text{reference}}/V_{\text{Doppler}}.
\]

Typically in clinical practice, when there is pulsatile flow, the velocity \(V_{\text{Doppler}}\) used in the Doppler continuity equation is the peak continuous-wave (CW) spectral Doppler velocity used in conjunction with a reference peak flow rate \(Q_{\text{reference}}\). Because CW Doppler records all velocities along its sample beam, and the highest velocities along a properly aligned sample beam through a stenotic orifice are thought to be at the vena contracta, the Doppler continuity equation is measuring the cross-sectional area of the vena contracta, called the effective orifice area (at peak flow), and not the true orifice area.

Often in clinical practice, the velocity \(V_{\text{Doppler}}\) used in the Doppler continuity equation is the temporal mean of the highest recorded CW velocities through a pulsatile cycle (in conjunction with a reference mean flow rate, \(Q_{\text{reference}}\)). These temporal means are generally obtained by measuring velocity-time integrals. In this case, the Doppler continuity equation is measuring a mean vena contracta cross-sectional area and not the true orifice area, because (1) the highest velocities along a properly aligned sample beam through a stenotic orifice are thought to be at the vena contracta and (2) the mean CW velocity is a temporal mean of the highest velocities recorded along that sample beam. Because of the constriction of flow at the vena contracta, this effective orifice area is generally smaller than the true orifice area (as in Fig 1) and reflects the actual area available for flow through the orifice.

The assumption of a uniform velocity through the cross section of the vena contracta (where the maximum velocity obtained by CW Doppler is assumed to be representative of the velocities over the entire cross section of the vena contracta) may not hold at certain flow states. If this assumption of a flat velocity profile is erroneous, inaccuracies will be introduced into the Doppler continuity equation.

Methods

Numerical Modeling

Numerical modeling flow experimentation consists of several stages: (1) grid generation, (2) specification of fluid properties and boundary conditions, (3) acquisition of flow solution, and (4) analysis of flow-data results. A software package from CFD Research was used in the numerical modeling experimentation. This software package was chosen because of superior qualities offered in (1) grid construction, (2) range of CFD solution schemes, (3) range of turbulence model selections, (4) visualization and analysis tools, and (5) technical support offered through CFD Research.
Grid Generation

Grid generation is the division of a flow domain (eg, a stenotic valve and the surrounding vessel structure) into a number of small nonoverlapping subdomains called finite control volumes. CFD-GEOM grid generation software was used to create all of our three-dimensional axisymmetric grids.

Flow Solution

Grids were fed into a finite-volume computational fluid dynamics (CFD) analysis package (CFD-ACE) by use of laminar and turbulence modeling. The finite-volume numerical algorithm consists of three steps: 1) integration of the governing equations of fluid flow (Appendix) over all the control volumes in the flow domain, conversion of these integral equations into a system of algebraic equations (called discretization), and solution of the algebraic equations by an iterative method called SIMPLEC. We used a second-order-accurate central differencing discretization scheme available in CFD-ACE. The performance of CFD-ACE in cardiovascular applications has previously been well documented.

A solution was considered proper once adequate convergence was obtained. At each iteration, CFD-ACE calculates a "residual" for each variable (variables defined in the governing equations of fluid flow, see Appendix) for all control volumes. A residual is the difference in value of a variable in a control volume from one iterative step to the next. Adequate convergence occurs when there is a reduction of four to five orders of magnitude in the maximum residual over all control volumes for all variables.

Adequate convergence also depends on the total mass imbalance in the system compared with the total mass flow rate through the boundaries of the domain. Mass flow rate through a surface is the product of fluid density, surface area, and the velocity normal to that surface. The total mass imbalance of a system is determined by subtracting the calculated mass flow rate through the inflow boundaries from the calculated mass flow rate through the outflow boundaries (in an ideal numerical solution, these should be equal). A total mass imbalance of three to four orders of magnitude less than the total mass flow rate through the boundaries is required for a solution to be considered proper. The CFD analysis package (CFD-ACE) used automatically calculates all of these parameters for analysis.

Turbulence Modeling

Turbulence modeling was done with the K Omega turbulence model (Appendix). Turbulence modeling was used for the simulations in which the Reynolds number at the orifice was >1000. Reynolds number ($N_{Re}$) is given as

$$N_{Re} = \frac{\rho V d}{\mu},$$

where $\rho$ is density, $V$ is velocity, $d$ is diameter, and $\mu$ is absolute viscosity.

Analysis of Flow Data Results

Flow results were analyzed with flow visualization software (CFD-VIEW).

Once a proper solution was obtained with each simulation, the location of the vena contracta was determined. The vena contracta was defined as the location downstream of the orifice at which path of a particle released from the outermost portion of the inlet side of orifice (ie, outermost streamline) was closest to axisymmetric axis.

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Figure 4. Velocity vector profile at vena contracta for 2.0-cm orifice at (a) low flow rate (6 mL/s, or 0.37 L/min) and (b) moderate flow rate (60 mL/s, or 3.7 L/min). Lengths of individual vectors represent magnitude of each velocity vector. To adequately display velocity profile differences, scales for vector length are not equivalent for (a) and (b). Note that shape of profile is parabolic for low-flow-rate case (a) and flattened and less parabolic in moderate-flow-rate case (b).

three-dimensional spatial average velocity in the vena contracta of the axisymmetric results for all runs.

Contraction Coefficients
The actual and Doppler-predicted contraction coefficients as a function of Reynolds number for all orifices were calculated. The actual contraction coefficient (CC(actual)) is defined by use of the recorded effective orifice area (OA_effective; equation 3) and the true orifice area (OA_true) as

\[ CC_{\text{actual}} = \frac{\text{OA}_{\text{effective}}}{\text{OA}_{\text{true}}} \]  

The Doppler contraction coefficient (CC(Doppler)) is defined by use of the calculated Doppler continuity equation orifice area (OA_DCE; equation 1) and the true orifice area (OA_true) as

\[ CC_{\text{Doppler}} = \frac{\text{OA}_{\text{DCE}}}{\text{OA}_{\text{true}}} \]  

Statistical Analysis
The actual and Doppler-predicted contraction coefficients were compared by standard paired Student’s t test. Values of P<.05 for paired t tests were considered significant.

Results
As an example of our results, velocity profiles from two separate flow rates through the 20-mm orifice are shown in

Figure 5. Percent difference between maximum and average velocities at vena contracta vs Reynolds number for 1.0-, 1.2-, 1.5-, and 2.0-cm-diameter orifice numerical model simulations. At low Reynolds numbers (<500), velocity profile is parabolic and difference between maximum and average velocities is greatest. At higher Reynolds numbers (>500), velocity profile flattens and difference between maximum and average velocity lessens (see also Fig 4).

Fig 4a and 4b (6 mL/s, or 0.37 L/min, and 60 mL/s, or 3.7 L/min, respectively). The velocity vectors shown are from a cross-sectional sampling through the vena contracta. Note that the velocity vector profile for the low flow rate (Fig 4a) is parabolic and the velocity profile at the moderate flow rate (Fig 4b) is flattened compared with Fig 4a. Note that with the higher flow rate, where there is a flat velocity profile at the vena contracta, the peak velocity and the average velocity will be similar in value, whereas with the low flow rate, where there is a parabolic velocity profile, the peak velocity and the average velocity will differ significantly.

In the results that follow, nondimensional Reynolds numbers (equation 2) are used to allow the identification of trends across all flow experiments, because the experiments performed involve a wide spectrum of orifice sizes and flow rates.

Fig 5 illustrates the concept of differences between average and peak velocities for all Reynolds numbers and orifices considered. Fig 5 shows the percent differences between maximum and spatial mean velocities across the vena contracta as a function of Reynolds number. The percent difference between the maximum and spatial mean velocities across the vena contracta was greatest at low Reynolds numbers (<500).

Fig 6 shows the actual and Doppler-predicted contraction coefficients as a function of Reynolds number for all orifices considered. Solutions using the laminar-flow model (with Reynolds numbers <1000) and the turbulent-flow model (with Reynolds numbers >1000) are shown. (Please refer to the “Limitations” section for a discussion of why the laminar and turbulent solutions are discontinuous.) P values for paired t tests are shown in parentheses in Fig 6, comparing results between CC_kinml and CC_Doppler. P value results for the laminar and turbulent models are shown separately; all values were P<.05.
Actual Contraction Coefficients:
Laminar-Flow Model
When the laminar-flow-model runs at low Reynolds numbers (<500) are used for all orifices considered (Fig 6), the actual contraction coefficients are in the range of 0.7 to 0.9, signifying that the effective orifice area underestimates but closely approximates the anatomic area of the orifice in low Reynolds number ranges. With moderate Reynolds numbers (500 to 1000), the actual contraction coefficients decrease, signifying that in this Reynolds number range, the effective orifice area decreases with increasing underestimation of true orifice area.

Actual Contraction Coefficients:
Turbulent-Flow Model
When the turbulent-flow model at high Reynolds numbers (>1000) is used for all orifices considered, the curve for the actual contraction coefficients as a function of Reynolds number (Fig 6) is nearly flat, remaining close to the value of 0.9, signifying that the effective orifice area remains close to and slightly less than the true orifice area at high Reynolds numbers.

Doppler Contraction Coefficient: Laminar- and Turbulent-Flow Models
As shown in Fig 6, at low Reynolds numbers (<500), the Doppler contraction coefficient significantly underestimates the actual contraction coefficient for all orifices considered. At moderate Reynolds numbers (500 to 1000), the Doppler contraction coefficient reaches its minimum; here it begins to track, while still underestimating the actual contraction coefficient for all orifices considered in the laminar-flow models. Throughout these low and moderate Reynolds number ranges (<1000), in the laminar-flow models, the Doppler contrac-
Flow Rate and Doppler Continuity Equation

Reynolds Numbers, Flow States, and Flow Rates

In the discussion that follows, we use modifiers to the terms "Reynolds number" and "flow state" interchangeably (eg, high flow state and high Reynolds number). If we examine the equation for the Reynolds number (equation 2), as the Reynolds number changes, orifice velocity changes proportionately if all other Reynolds number parameters are held constant, including orifice size. Qualitatively, at least, flow state changes occur proportionately to changes in velocity.

Observations based on a dimensionless parameter such as the Reynolds number allow the identification of certain trends across flow experiments, experiments that include a wide spectrum of orifice sizes and flow rates. However, the ranges of flow states or Reynolds numbers discussed (ie, low, moderate, high) were based on analysis of our results, and these ranges are not immediately applicable to cardiac output flow rates used clinically. The Table gives flow rates for low, moderate, and high Reynolds number ranges for each orifice considered.

Discussion

The Doppler continuity equation has become a popular noninvasive means of assessing stenosis severity. This present investigation was designed to study, in isolation, the various factors that may influence differences between the true orifice area, the effective orifice area, and the orifice area calculated by the Doppler continuity equation in stenotic orifices. Results from our numerical modeling indicate that (1) the effective orifice area underestimates true orifice area; (2) the effective orifice area is flow dependent (ie, it changes with differing flow states) regardless of whether the true orifice is flow dependent, although the effective orifice area remains fairly constant at high Reynolds numbers; (3) the Doppler continuity equation underestimates the effective orifice area and the true orifice area throughout all conditions considered; and (4) the Doppler continuity equation is flow dependent, with the greatest underestimation of effective orifice area at low Reynolds numbers (<500) and the greatest underestimation of true orifice area at moderate Reynolds numbers (500 to 1000).

Effective Orifice Area

Several investigators have explored the relationship between effective orifice area and flow rate. However, all of these previous studies have relied on the Doppler continuity equation or the Gorlin and Gorlin formula to calculate effective orifice area, both of which may have other inherent dependencies on flow rate. Numerical modeling provides a "gold standard" method of predicting effective orifice area.

Our results demonstrate that effective orifice areas (ie, vena contracta cross-sectional areas) underestimate but approach true orifice areas at low (<500) and high (>1000) Reynolds numbers, corresponding to low- and high-flow states in these experiments. At moderate Reynolds numbers (500 to 1000), the effective orifice area narrows to 70% to 75% of true orifice area in our model. A probable explanation is that at low Reynolds numbers, the velocity and thus the momentum of the proximal streamlines converging into the orifice from the sides is decreased. This reduction in momentum allows these flow streamlines to change direction more rapidly when they encounter the main flow stream without causing a significant reduction in distal flow area. Consequently, the vena contracta region for such low-flow states would occupy almost the entire true orifice area. At moderate Reynolds numbers, the effective orifice area narrows because of the increased momentum of the proximal streamlines converging into the orifice from the sides. At high Reynolds numbers, turbulence blunts this constricting effect and the effective orifice area increases once again, thereby approaching the true orifice area.

Vena Contracta Velocity Profile

Our results show that the velocity profile at the vena contracta is not flat, especially for low Reynolds numbers (corresponding to low-flow states in these experiments), at which it assumes a more parabolic profile (Fig 4a). At moderate Reynolds numbers (500 to 1000), corresponding to moderate-flow states in these experiments, the velocity profile flattens out (Fig 4b). Fig 5 shows graphically the percent difference between maximum and mean velocities at the vena contracta relative to Reynolds numbers.

A likely explanation for changes in the velocity profiles with differing flow states comes from the flow dependence of viscous and turbulent effects. At low-flow states, viscous effects from the sides of the orifice retard outside velocities while centerline velocities increase to maintain flow-rate
equity, resulting in a velocity profile that diverges from flat at the vena contracta. At moderate- and high-flow states, viscous effects diminish, allowing turbulence effects to play a more important role. Velocity profiles in turbulent flow are known to be inherently blunt and flat, and our results confirm this. To the best of our knowledge, no previous investigations have studied the actual velocity profiles across the vena contracta in stenotic lesions.

**Doppler Continuity Equation**

Our results elucidate two fundamental reasons why the Doppler continuity equation underestimates true orifice area and is flow dependent. First, because of the vena contracta effect, the Doppler continuity equation theoretically measures the effective orifice area (see “Theoretical Considerations”). As we have demonstrated, the effective orifice area itself underestimates true orifice area and is flow dependent. Second, the Doppler continuity equation does not accurately track the effective orifice area. The inaccuracies in tracking the effective orifice area can be explained by the fact that the maximum velocity obtained clinically with CW Doppler and used in the Doppler continuity equation is thought to be representative of the spatial average of velocities over the entire cross section of the vena contracta. This holds true only in the presence of a flat velocity profile throughout the vena contracta. Any variation from a flat velocity profile will lead to overestimation of the spatial average of velocities and consequently to an underestimation of orifice area in the Doppler continuity equation, because velocity is in the denominator (see equation 1).

At low Reynolds numbers (<500), corresponding to low-flow states in these experiments, the effective orifice area underestimates the true orifice area and the vena contracta velocity shows a parabolic nonflat profile. Both of these effects (especially the parabolic velocity profile) cause the Doppler continuity equation to underestimate the true orifice area.

At moderate Reynolds numbers (500 to 1000), corresponding to moderate-flow states in these experiments, the effective orifice area begins to succumb to constriction effects, and there is increasing underestimation of true orifice area. The vena contracta velocity profile is flatter. Thus, errors from velocity profile effects become minimal and changes in the effective orifice area dominate, causing the Doppler continuity equation to continue to underestimate the true orifice area.

At high Reynolds numbers (>1000), corresponding to high-flow states in these experiments, the effective orifice area increases because of turbulence effects, and the vena contracta velocity profile continues to be flat. Thus, errors from both of these effects are minimized, and there is less underestimation of true orifice area by the Doppler continuity equation.

**Clinical Implications**

The flow rate dependence of the Doppler continuity equation has been explored by investigators in the clinical setting by exercise or dobutamine stress echocardiography in aortic stenosis. Most of these studies demonstrate changing Doppler valve areas with varying cardiac outputs. Investigators who have shown an increase in Doppler valve areas with increasing flow theorize that the valve leaflets open farther when exposed to the greater pressure gradient. Although this may undoubtedly be part of the explanation, our studies on rigid orifices offer an additional reason for the Doppler underestimation of effective orifice area at low cardiac outputs, which is inherent in the assumptions of the equation itself. These assumptions must be kept in mind when we apply Doppler methods in clinical practice. If treatment decisions are to be based on the measurements derived from the Doppler continuity equation, understanding these flow-rate dependencies becomes quite important.

**Limitations**

With our numerical experimentation tools, we performed detailed investigations of the flow dynamics through stenotic orifices, which provided a unique means of assessing the accuracy of the Doppler continuity method. Although the fixed rigid orifices in our model are admittedly a simplification of a complex three-dimensional structure of a stenotic valve, they allow isolation of the effects of flow on vena contracta or effective orifice area without the added complexity of possible anatomic expansion of a physiological orifice as flow rate increases.

Numerical modeling was used because it provides a unique medium for detailed examination of the fluid dynamics around the vena contracta region. The limitations of all numerical models include the fact that a selected number of numerical solutions should be validated with in vitro models. Preliminary in vitro studies in our laboratory show similar results for the deviation of the velocity profile away from flat to more parabolic profiles across the vena contracta at low-flow states as well as changes in effective orifice area from low- to moderate-flow states. Work is in progress comparing the velocity vector fields of the numerical and in vitro models for further validation of our method.

Numerical modeling over such a wide range of Reynolds numbers presents unique problems for any CFD algorithm. Our ranges of Reynolds numbers include modeling laminar flow, transitional flow, and turbulent flow. The decision to turn on turbulence modeling at Reynolds numbers >1000 was done with some insight as to when turbulence occurs in these types of problems. However, selecting this turbulence threshold was still somewhat arbitrary.

A discontinuity between laminar and turbulent model contraction coefficients (Fig 6) occurs at this turbulence threshold. We believe this represents a weakness of the CFD algorithm we used in predicting results in the transitional flow range. To the best of our knowledge, this weakness is shared by all CFD algorithms developed to date. A weakness of a CFD algorithm in predicting results in this transitional flow range does not negate its results in the laminar and turbulence ranges. Such a sharp discontinuity in the contraction coefficient curves probably does not occur in reality. It is more likely that there is a much smoother transition from one curve to the other in the transitional-flow Reynolds number region (~1000). Overall, however, the finite-volume CFD analysis package (CFD-ACE) we used provides a robust solution algorithm, as suggested by the agreement with our in
vitro validation studies to date. A complete description of the transition from laminar- to turbulent-flow processes is beyond the scope of this investigation; for further discussion, the reader is referred to Landahl et al. Our definition of the vena contracta relies on determining streamlines and particle-tracking algorithms. How well these algorithms predict the average path of a particle in turbulent flow is a question that we will be addressing in future studies.

Our model considers steady flow. Pulsatile-flow numerical runs were prohibitive in computing-power and time requirements to obtain solution convergence practically. A relatively dense grid pattern in our model was required for the detail that was necessary in examining the vena contracta region. Dense grid patterns require relatively small time increments in solving for pulsatile flow. Unfortunately, to obtain proper numerical solutions, dense grid patterns and small time increments require computational power beyond our present capabilities. With this, we elected to run the required dense grid patterns under steady-flow conditions.

We considered flow rates (6 to 400 mL/s, or 0.37 to 25 L/min) that allow for comparison to mean and peak flow rates seen clinically. Strictly speaking, simplifying the analysis of pulsatile flow by comparing conditions at peak or mean flow rates in a pulsatile system with steady-flow conditions at equivalent flow rates is not entirely accurate. Thus, these steady-flow experiments do not allow for the direct assessment of the ability of the Doppler continuity equation method to measure the effective orifice area using peak or mean flow rates and velocities under pulsatile conditions. However, these steady-flow experiments have allowed us a unique window into the complexities of the fluid dynamics near a stenotic orifice. Further pulsatile-flow studies will be required. In preliminary numerical and in vitro pulsatile-flow experiments in our laboratory, similar trends in velocity profiles versus Reynolds number have been noted.

Extending our results to the aortic valve does not account for the effects of the aortic walls, which are much closer to the stenotic orifice than in our model. In addition, we have not examined errors introduced into the clinically applied Doppler continuity equation as a result of calculations involved in the determination of the reference flow rate (see equation 1), in which this reference flow rate \( Q_{\text{reference}} \) is typically determined by use of spectral Doppler measurements.

Conclusions

Our results demonstrate the flow dependency of the Doppler continuity equation. This flow dependency is due to flow-dependent changes of the effective orifice area (cross-sectional area of the vena contracta) as well as to changes in the vena contracta velocity profile. At low-flow states, the velocity profile is skewed, causing underestimation of true orifice areas. At moderate-flow states, the velocity profile becomes more flattened, lessening its effects on orifice area errors; however, the vena contracta (effective orifice area) becomes more constricted, causing continued underestimation of true orifice areas. At high-flow states, the velocity profile remains flattened, and there is less constriction at the vena contracta (ie, greater effective orifice area), causing less underestimation of true orifice areas.

The clinical implications of our study are that (1) the underestimation of true orifice area at low cardiac output states by the Doppler continuity equation may lead to erroneous clinical assessments and unnecessary therapeutic interventions, and (2) results of clinical studies by other investigators that have shown an increase in the Doppler continuity equation-calculated valve area with exercise or dobutamine infusion can be explained by fundamentally sound fluid dynamic principles that compound the generally accepted explanation of increase in valve leaflet opening.

Appendix

The set of governing equations for fluid flow consists of the equations for conservation of mass (continuity equation) and conservation of momentum (Navier-Stokes equations). We consider blood to be an incompressible Newtonian fluid for which the equations for conservation of mass (equation 6) and conservation of momentum (equations 7 through 9) become (in Cartesian coordinates)

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \\
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right).
\end{align*}
\]

Here, \( u, v, \) and \( w \) are the velocity components in the \( x, y, \) and \( z \) directions, \( p \) is pressure, \( t \) is time, and \( \nu \) is kinematic viscosity (absolute viscosity/density). The CFD analysis package (CFD-ACE) solves for the variables \( u, v, \) and \( w \) in laminar-flow situations.

Turbulent flows are inherently unsteady and contain wide ranges of time and length scales. Thus, in turbulent-flow situations, CFD-ACE uses equations derived from averages of the governing equations over time to yield Favre or density-averaged equations. Using Favre averaging introduces additional terms, known as Reynolds stresses, in the governing equations listed above. CFD-ACE solves for terms related to these Reynolds stresses as well as the variables \( u, v, \) and \( w \) in turbulent-flow situations.

It is beyond the scope of this article to describe the various turbulence models available or the details of the K Omega turbulence model selected; the reader may refer to the references noted. The K Omega turbulence model was chosen because it is a "low Reynolds turbulent number" turbulence model, since it permits integration of the momentum equations and the turbulence equations all the way to the model walls, which was necessary because of the grid resolution required near the wall of the orifice.

The term "low Reynolds turbulent number" does not refer to the standard Reynolds number (equation 1). The standard Reynolds number represents a ratio between inertia (momentum) forces and viscous (friction) forces. A low Reynolds turbulent number refers to a qualifier on the Reynolds number turbulent model (\( N_{\text{Re}} \)):

\[
N_{\text{Re}} = \rho \sqrt{\frac{\kappa}{\mu}} \frac{\epsilon}{\mu},
\]

where \( \kappa \) is turbulent kinetic energy, \( \epsilon \) is rate of dissipation of kinetic energy, \( \rho \) is density, and \( \mu \) is absolute viscosity. The \( N_{\text{Re}} \) represents the ratio between eddy viscosity and molecular viscosity. Molecular viscosity relates shear stress to the strain rate. Eddy viscosity relates Reynolds (turbulent) stresses to the mean strain rate.
The K Omega turbulence model requires that the first grid point from a wall must be placed in the laminar sublayer. The laminar sublayer is defined by a location in the grid at which the distance from the wall specifies a value of a dimensionless number, \( y^+ \), between 0 and 5. \( y^+ \) is a dimensionless number at the walls, defined as:

\[
y^+ = \frac{\rho u_1}{\mu},
\]

where \( y \) equals the distance to closest grid point from the wall and \( u_1 \) is the friction velocity \([U^{1/2}\tau]\), where \( \tau \) is wall shear stress. CFD-ACE automatically calculates \( y^+ \) (equation 11) at the walls.

To ensure that the first grid point from any wall was in the viscous sublayer (for those runs requiring turbulence modeling), the grid was checked (and modified if necessary) for each run so that the value of \( y^+ \) at the walls was between 0 and 5.\(^{15,16}\)

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**References**


Analysis of the Effect of Flow Rate on the Doppler Continuity Equation for Stenotic Orifice Area Calculations: A Numerical Study
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