Electrocardiographic Leads

III. Synthesis

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This concluding paper of a series of three on electrocardiographic leads indicates how leads that should be superior to many now in use may be constructed. The design of leads to obtain accurate representations of the mean cardiac vector in the usual planes of reference, methods for the improvement of several of the conventional leads, and a unipolar lead of a new type are emphasized.

In the first and second papers of this series1,2 the concept of the “lead field” was introduced, and it was shown how the behavior of leads may be determined by an analysis of their lead fields. In this final paper, methods for the construction of leads that should be superior to those in common use are described.

Although some of these methods are complicated, and to a certain extent mathematical, their purpose is practical. They should result in leads of exceptionally high accuracy. For example, when the precision of the standard leads is not adequate, these methods show where electrodes may be placed and how they may be interconnected so that greater accuracy is attained. In cases where unipolar electrocardiograms are being taken and the exploring electrode is on the back or sides, rather than on the anterior chest, the methods show how the central terminal may be modified so that it remains indifferent. Again, using these methods, a lead can be built which yields an accurate measure of the sagittal component of the heart vector.

These methods for synthesizing leads are variations of a single basic procedure which is as follows. First, the exact nature of the desired lead is specified by describing what its field in the heart ought to be. This is usually done by modeling the lead after a “prototype” lead in an infinite homogeneous conductor. Second, a lead is constructed which actually produces such a field, or a close approximation to it, taking into account the irregular shape and conductivity of the human body.

Two basic methods for the design of leads are presented. The first and more practical approach relies on the combination of several leads in such a fashion that undesirable aspects of the field, such as incorrect general direction, flare and curvature are systematically cancelled out. This method (“synthesis of leads by combination”) is essentially a generalization of the methods for correcting the direction of lead vectors which have already been worked out by Burger and van Millaan,3 by Wilson, Bryant and Johnston,4 and by Beeking, Burger and van Millaan.5

The second more mathematical approach attempts to determine the precise nature of the lead field on the body surface which would produce the correct field in the heart. This surface field is then approximated by connecting surface electrodes to the two terminals of the lead through suitable networks of resistors. The success of this second method (“synthesis of leads by adjustment of the lead field current at the body surface”) is due to the fact that it is mathematically much easier to find the surface currents necessary to produce a specified field in a conductor than it is to find the field within the conductor resulting from specified surface currents.

The first part of the paper, which deals with the first approach, begins with a procedure for obtaining a set of three terminals whose lead vectors form an Einthoven triangle; that is to say, three terminals whose three leads produce fields at the center of the heart having the same direction that the fields of the standard leads would have if all of Einthoven’s assump-
tions were correct. Following this, the construction of a simple lead for obtaining all three components of the heart vector from a set of three independent leads with one common electrode is discussed. Next, means of correcting the curvature of the fields of lead I and the central terminal chest leads are taken up.

In the second part of the paper the general idea of the second approach is illustrated by a qualitative example which shows how any lead for obtaining the sagittal component of the heart vector must be constructed. Then a method is presented for the construction of "perfect" heart vector leads for a "body" which is a homogeneous box, and it is indicated why this same general procedure may be employed to obtain such leads when the body is assumed a homogeneous conductor of any shape containing a spherical or ellipsoidal heart of greater conductivity than its surroundings.

In the latter portion of this part of the paper similar methods are applied to the problem of the construction of a perfect "indifferent electrode." It is shown here that it is theoretically possible to obtain a type of unipolar lead for which the effective location of the exploring electrode is within the body, close to the surface of the heart, despite the fact that all of the electrodes of the lead are actually on the body surface.

The third part of the paper discusses limitations to the synthesis of leads. Here, a simple demonstration shows that it is impossible to construct a lead whose voltage is influenced by the electromotive force (EMF) within a single small region of the heart. This emphasizes the extremely important fact, pointed out by numerous investigators, that one cannot get a completely detailed picture of the electromotive forces of the heart from measurements of the potentials outside of it. Another problem that is considered is the possibility of constructing a lead with a perfectly uniform or radiating field in a conductor (such as the body) whose conductivity variations are not of a simple type.

![Diagram](https://example.com/diagram.png)

**Fig. 1.** (a) Burger triangle for standard leads. (b) Network for transforming it into an Einthoven triangle.

### Synthesis of Leads by Combination

**Three Terminals whose Lead Vectors Form an Einthoven Triangle**

Experiments with cadavers and normal subjects, with electrolytic models, fluid mappers and conducting paper manikins, all indicate that at the center of the heart the Burger triangle has more or less the shape* shown in figure 1a. Assuming the accuracy of this, it is possible to find three terminals, connected to the electrodes of the arms and left leg by resistors, which are associated with an equilateral Burger triangle. A graphic procedure for doing this is illustrated in figure 2. In step 1 of this figure a line is drawn horizontally from R until it intersects the opposite side at point \(L'\). The distances between this point, \(L'\), and the two ends, \(L\) and \(F\), of the line determine the ratio of the two resistances \(r_1\) and \(r_2\), as shown in step 2. In step 3 a compass is used to find the point \(F''\), which completes the equilateral triangle having the line \(RL'\) as its top. A straight line drawn through \(R\) and \(F''\) extended to the opposite side, \(LF'\), determines an intersection

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* The exact dimensions of the triangle were so chosen that they are the same as those found by Burger and vanMilaan. (Reference 2, part II, page 157.)
point, P, and in step 4 this point is obtained electrically with resistors $r_3$ and $r_4$. The addition of the fifth resistor, $r_5$, in step 6, in accordance with measurements of $d_5$ and $d_6$ made in step 5 produces the desired third terminal, $P'$.

This procedure involves an important rule which applies when one knows the lead vectors of several electrodes (or terminals) with respect to a common or reference electrode, and wishes to find the vector associated with a tap on a resistor (of a relatively high
value) connected between two of these electrodes. This rule, which was pointed out both by Wilson, Bryant and Johnston and by Burger and van Milaan, states that the tap will have the same position on a straight line joining the tops of the vectors associated with the two electrodes as it has on the resistor. This justifies, in figure 2, the relations

\[
\frac{r_1}{r_2} = \frac{d_1}{d_2}, \quad \frac{r_3}{r_4} = \frac{d_3}{d_4}\left(1 + \frac{1}{r_3 + r_4}\right) = \frac{d_5}{d_6} \quad (1)
\]

A proof of this important principle is given in the Appendix. Note that in step 6 the parallel resistance of \(r_3\) and \(r_4\) had to be taken into account.

One should also note that the connection of resistor \(r_4\) between \(R\) and \(F'\) will change the magnitude but not the direction of the lead vector. The effect of the added resistor, of course, is simply to shunt off some of the lead field current which would otherwise pass through the body.

The total circuit is that shown in figure 1b. Here the resistors have been made sufficiently large so that the leads will be insensitive to variations of contact resistance, and will have negligible influence on the fields of the body. The average length of these three new lead vectors can be made equal to the average length of the vectors of the original triangle by increasing the sensitivity of the electrocardiograph from 1 cm. per millivolt to 1.91 cm. per millivolt. This will keep the average voltage deflection produced by the three leads the same as it was before.

**Obtaining the Components of the Heart Vector from a Set of Four Electrodes**

In the preceding section we have seen how the direction of the lead field at the center of the heart may be adjusted through the use of external resistors. This example, however, was a two dimensional one. Let us now consider how the same thing may be done with the three components of the lead field using a set of four electrodes.

Suppose, for example, that experiments have been carried out by methods like those discussed in a previous paper of this series\(^\dagger\) to find

the exact values of the lead fields at the center of the heart for the following three leads; RF, LF and SF, where \(R\) is the right arm electrode, \(L\) the left arm electrode, \(F\) the left foot electrode and \(S\) is a fourth electrode whose voltage is influenced considerably by the sagittal component of the electromotive forces.\(\ast\) Suppose further that these experiments yield the following results.

\[
\begin{align*}
\vec{J}_{FR} &= 8 \vec{I}_V + 4 \vec{I}_T + \vec{I}_S \\
\vec{J}_{FL} &= 12 \vec{I}_V - 8 \vec{I}_T - 2 \vec{I}_S \\
\vec{J}_{FS} &= 8 \vec{I}_V - 2 \vec{I}_T - 16 \vec{I}_S
\end{align*}
\]

where \(\vec{J}_{FR}, \vec{J}_{FL}, \vec{J}_{FS}\) are the lead field current densities associated with leads \(FR, FL,\) and \(FS,\) and \(\vec{I}_V, \vec{I}_T,\) and \(\vec{I}_S\) are unit vectors in the vertical, transverse, and sagittal directions respectively. How can one combine these three leads to produce a fourth lead whose lead field at the center of the heart is directed in any one of these reference directions?

The first thing to do is to find a combination of the three leads which cancels out the two undesired components of the lead field at the center of the heart. This can be done as follows. First, find two combinations each involving a pair of leads for which one of the undesired components has been eliminated. For example, in this case

\[
\begin{align*}
\vec{J}_{FR} - \vec{J}_{FS} &= 6 \vec{I}_T + 17 \vec{I}_S \\
\vec{J}_{FL} - \frac{3}{2} \vec{J}_{FS} &= -5 \vec{I}_T + 22 \vec{I}_S
\end{align*}
\]

Here the vertical component \(\vec{I}_V\) has been removed. Next, combine these two in such a fashion that the remaining undesirable component is cancelled out; that is

\[
5 (\vec{J}_{FR} - \vec{J}_{FS}) + 6 (\vec{J}_{FL} - \frac{3}{2} \vec{J}_{FS})
= 217 \vec{I}_S = 5 \vec{J}_{FR} + 6 \vec{J}_{FL} - 14 \vec{J}_{FS}
\]

This equation states that if a current of 5, 6 and \(-14\) amperes\(\dagger\) enters the RF, LF and SF leads respectively, then the resultant field at the center of the heart will be directed in an

\(\ast\) The nature of this electrode is described later in this paper, page 873.

\(\dagger\) This could just as well be milliamperes, the units are not important.
anterior-posterior direction. Under these circumstances the current entering the body through the $R$, $L$, $S$ and $F$ electrodes, taken by themselves, will be 5, 6, $-14$ and 3 amperes respectively. The problem now is to design a resistor network which, when connected to a single source of current, will cause currents having these same relative magnitudes to enter the four electrodes $R$, $L$, $S$ and $F$, since it is clear that the voltages produced in such a lead by EMF's at the center of the heart will be proportional only to the sagittal components of those EMF's.

The procedure involved here is a simple one. Electrodes carrying current which leaves the body, in this case $S$, are connected to the positive terminal of the lead, and electrodes having current entering them, here $R$, $L$ and $F$, are connected to the negative terminal of the lead. The size of the resistors is adjusted so that the current splits up into the desired proportions. To do this, all resistors are first made large enough so that the potential produced between any two electrodes on the body by the lead field current is negligible compared to the voltage drop across the resistors themselves. When this is done, all the electrodes will have essentially the same potential, and the situation can be represented electrically by the diagram shown in figure 3a.

This diagram makes the method for adjustment of the current values quite apparent. Take, for example, the resistors between the negative terminal of the lead and the common junction representing the body. The voltage across all three of these resistors will be the same, and thus the current through each will be, by Ohm's Law, inversely proportional to the resistance of each. Thus, to make the current flowing into the $R$ electrode $\frac{3}{5}$ times as great as the current flowing into the $F$ electrode, the resistance connected to the $R$ electrode must be equal to $\frac{3}{5}$ that of the $F$ electrode. In a similar way the resistance of the $L$ electrode is made equal to $\frac{1}{2}$ of the resistance of the $F$ electrode. The complete circuit for obtaining the sagittal component of the electromotive forces, then, is shown in figure 3b. This figure also shows the circuits for obtaining the vertical (c) and transverse (d) components of the electromotive forces. Note that in these last cases two and four, rather than three resistors are needed.

In general, the procedure may be summarized as follows.\(^*\)

(1) Find the combination of the leads which produces the desired field.
(2) From this find the value of the field current flowing into or out of each electrode.
(3) Connect, through resistors, all the electrodes with outflowing current to the positive terminal of the lead, and all electrodes with inflowing current to the negative terminal of the lead. Make the relative conductances of each resistor equal to the relative intensities of the current flowing in or out of the electrodes associated with them.

This procedure is a very basic one. All methods of synthesizing leads discussed in this paper, regardless of the number of electrodes

\(^*\) A similar procedure has been worked out by Burger and his associates.
involved, depend on it. Note in particular that the procedure will succeed with any combination of leads whatsoever. This means that any linear combination of leads can be obtained with a resistor lead and one amplifier. This is important because in most cases it is more practical to combine the leads before rather than after amplification.

**Correction of Curvature and Flare**

Just as the error in the general direction of a field may be corrected by adding to it other fields which cancel out the undesirable component, so also may undesirable or excess curvature or flare of field be eliminated with the addition of the fields of another lead.

This is illustrated by the example in figure 4, which shows a sketch of the field of lead I before and after correction. The addition of the transverse lead made up from electrodes on the sides below the level of the heart serves to cancel out the curvature of lead I, since the field of the lower lead has the opposite curvature. It might be noted that the errors in general direction of the two leads also tend to cancel each other.

Another example is given in figure 5, which shows a central terminal chest lead before and after correction. In this case an electrode on the back was added. The excess flare toward the posterior side of the heart is cancelled out by the deficiency in the flare in the lead which is combined with the first one.

Simultaneous correction of the general direction and the curvature of the lead field within the heart, especially if the field is to be corrected at two or more points, may become rather complicated. Using the principles outlined above and a larger number of suitable leads, the desired field can be closely approximated either by a mathematical procedure or by an experimental method of successive approximations. A detailed discussion of this matter is beyond the scope of this paper.

**Synthesis of Leads by Adjustment of Lead Field Current at the Body Surface**

In combining leads in such a fashion that undesirable aspects of the fields cancel out, there will always remain small imperfections in the result. By combining more and more leads it may be possible to cancel out even these small imperfections until, through the use of an infinite number of leads, the resultant field becomes exactly that desired. However, when this is done, it will be found that the lead field current is passing into and out of the body through electrodes covering its entire surface. That is to say, a perfect lead requires in general a surface current “distribution” of a certain specific type. For a given lead field in the heart, then, what is this current distribution?

**A Simple Sagittal Lead**

As an elementary and practical illustration of the idea involved here, consider the construction of a simple lead for obtaining, in the best possible manner, the sagittal component of the mean heart vector.

Looking at the body from the side, the flow lines of the field of a perfect sagittal lead should have the pattern shown in figure 6a. To
obtain a lead field of this kind it is nearly self evident that a system of multiple electrodes located on the left posterior chest behind the heart through which currents enter and a similar set of electrodes on the precordium from which they pass out will be required.

Such an arrangement is shown in figure 6b. Here five electrodes are placed on the chest over the heart, and each of these electrodes is connected through a large resistance (for example, 20,000 ohms) to the positive terminal of the lead. The negative terminal of the lead is attached to a similar set of electrodes on the back directly behind the heart.

The positive terminal of this lead is the "S" terminal which was previously used, along with the \( R \), \( L \) and \( P \) terminals, to produce lead fields at the heart center (fig. 3). So far as the external networks are concerned, this may be thought of as a single electrode in series with a resistor whose value is one-fifth of \( r \). This resistance must be taken into account in the construction of the networks shown in figure 3.

Obviously, the greater the number of electrodes used in front of the heart, the more regular the field there will be. One way to test the accuracy of using a fairly small number of electrodes is to compare the actual electrocardiographic voltages obtained this way with the voltages obtained from a lead using a large number of electrodes.

**The Homogeneous Box Model**

It is possible to get insight into the surface current distributions required of perfect leads by considering a simple yet not inaccurate model of the body, a box shaped conductor of uniform conductivity. In order to agree with the average dimensions of the human torso, the box should be about half as wide and two-fifths as deep as it is long.

It is not difficult to see what the surface current distribution should be in this case when it is desirable to construct leads for determining various components of the heart vector. If a uniform amount of current enters all points on the top of this box, and leaves it at the bottom, and no current enters or leaves the sides, then it is evident from the symmetry of the situation that the field of current at all points within the box will be uniform and directed perpendicularly. This, of course, means that a lead producing such a surface current distribution on the top and bottom of the box will be sensitive to the vertical component of the heart vector.

To construct such a lead practically, one connects electrodes covering the areas where the current enters or leaves to the terminals of the leads through resistors (fig. 7a). As before, the resistors should be so large that the potential drop between the terminals is negligible in comparison to that across the resistors.

The other components of the heart vector are obtained in a similar manner. It might be noted that since the resistors connected to each side are alike, the potential of the terminal of the lead connected to a side will be the average of the potential of that side. Thus, we have just shown that the difference between the average potentials of two opposite sides of a homogeneous box-shaped conductor will be proportional to the components of the dipole moment of the electromotive forces perpendicular to those sides.

The proportionality factor depends upon the area of the sides through which the current is entering and leaving. If that area is small, the current density will be large, and if the area is large, the current density will be small. The fields of all three leads can be made equal in intensity by putting a shunt resistor of suitable value between the terminals of the
leads attached to the two pairs of sides having relatively small cross-sectional areas.

Leads of a similar kind may be used to obtain desired components of the mean cardiac vector if the body is assumed to be a homogeneous conductor of arbitrary shape or a conductor of irregular shape with a spherical or ellipsoidal heart of greater conductivity than other tissues. The latter corresponds closely with the situation that exists in the living human body. The procedure may be outlined as follows. First, assume that the lead field in the heart is uniform and pointed in the desired direction. Next, determine analytically what the field must be outside the heart, particularly its perpendicular component of current at the body surface. Lastly, construct a lead which introduces at every point on the body surface the same amount of current. It follows from the well known “uniqueness” theorem of electrical theory that this lead will produce the correct field in the heart.

In order to make the value of the lead field at the body surface exactly equal to the desired value, one would need an infinite number of resistors attached to an infinite number of minute electrodes covering the entire body surface. As a practical thing, this is obviously impossible, and also unnecessary. If a fairly small number of electrodes is used, there will be irregularities in the fields right at the surface, but these irregularities will all but disappear so far as the field within the heart itself is concerned. Only when the surface is very close to the heart and the amount of current entering or leaving there is fairly substantial need one be careful concerning this. (The lead for obtaining the sagittal component of the heart vector is a good example of this situation.) In other cases one can simply keep the total conductance (per unit area) of the attached resistors constant and the fields will smooth themselves out as they approach the heart. The best criterion for the number of electrodes necessary is comparison of voltages obtained with a large (and clinically impractical) number, and those obtained when a smaller number of electrodes is employed. When a small number of electrodes furnishes voltages nearly identical to those obtained with the larger number, the smaller number will be sufficiently accurate.

Unipolar Leads

Suppose that the electromotive forces of the heart were removed from the body and placed in an infinite homogeneous conductor. They would set up an electric field producing at every point a certain potential relative to an
electrode at infinity (or an equivalent indifferent electrode). Let us now attempt to find the lead field current distribution on the body surface required of a real lead in order that it have the same potential difference induced in it.

Two different situations must be considered. In the first and simpler of the two, the exploring electrode is located on the surface of the body and is to have a lead field identical to that of an exploring electrode in a similar position in the infinite homogeneous conductor. The problem here is to obtain a satisfactory indifferent electrode. In the second case, however, the effective location of the exploring electrode is no longer restricted to the surface of body. Here a "phantom" exploring electrode is constructed which acts as if it were located at any specified point outside the heart, although all electrodes of the lead are actually located on the skin. Leads of this type might have clinical application in the location of posterior infarcts and other localized lesions.

Consider first the simpler approach, where the exploring electrode is at the body surface. If the body is assumed to be homogeneous, it is not difficult to calculate the surface current which the indifferent electrode must produce in order that the current radiate from the exploring electrode in straight lines. This is done by assuming that the radiating field already exists and determining the amount of current passing into and out of the body at all points on its surface. The procedure is similar to that used to construct heart vector leads in the case where the body was assumed to be homogeneous but of arbitrary shape.

The second case, involving the phantom electrode, makes use of the following fact. The desired field of the lead, which is based on the prototype field in the infinite homogeneous conductor, can be expressed as an infinite series, in which each term is a "spherical harmonic"; i.e., a product of a power of the distance from the center of the heart and a Legendre polynomial. For example, in figure 7b, if a unit current enters at A and streams out to infinity, then the potential of the lead field between B and infinity is given by

\[
\phi = \frac{1}{4} \pi \rho = \frac{1}{4} \pi \rho (r^2 + d^2 - 2rd \cos \theta)^{\frac{1}{2}}
\]

or

\[
\phi = \left( \frac{3}{4} \pi d \rho \right) \cdot \left[ 1 + \left( \frac{r}{d} \right) P_1 \cos \theta + \left( \frac{r}{d} \right)^2 P_2 \cos \theta + \cdots \right]
\]

The \( P_n \) \( \cos \theta \) coefficients here are the associated Legendre functions and \( \rho \) is the conductivity of the medium. This mathematical expression (6) is valid only when \( r \), the distance from the center of the heart to the point \( B \), where the field is measured, is less than \( d \), the distance from the center of the heart to the location \( A \) of the exploring electrode. When \( r \) is greater than \( d \), the sum becomes infinite.

The sum of the terms of (6) gives a simple radiating field with straight flow lines. But each term of (6) separately obeys Laplace's equation (in a homogeneous medium), and this describes a field. In order to emphasize this point, the two dimensional analogues to the second, third and fourth terms are sketched in figure 8. (The second term here is clearly the uniform field characteristic of "heart vector" leads.) Thus, the field of an exploring electrode may be thought of as the superposition of an infinite number of component fields of increasing complexity and diminishing intensity.

The values of these component fields at the body surface can be used to determine surface current distributions capable of generating each component. If the distributions associated with the first few terms of series (6) are added together, the resultant field will be the same field given by equation (6) less the field due to the sum of the terms of the series not included. Since the series is a power series which converges quite rapidly,* the effect of these neglected terms will be small, and so the synthesized field will in general be all but identical with the desired field within the heart.

* Provided that the exploring electrode is not very close to the surface of the heart.
Thus, in order to construct a lead where the location of the exploring electrode is to be within the body, close to the heart, the following procedure can be used: (1) Find the spherical harmonic expansion of the exploring electrode field in the heart using the infinite homogeneous conductor model. (2) Find the current distributions on the body surface capable of producing the field associated with each of the first few terms of the expansion. (3) Add these current distributions and find the resultant. (4) Construct a lead producing a surface field approximately equal to this resultant.

Of course, the close correspondence between the field of the exploring electrode in the infinite homogeneous conductor and the field in the body exists only within the "heart" region. Outside the heart, and particularly outside the circle centered on the heart and passing through the "phantom" exploring electrode, the field in the body bears no resemblance to the field which actually would be produced by a real exploring electrode at that point.

Another important fact is that, as the number of terms being used increases, the intensity of the field inside the heart, as compared to the field outside it, becomes smaller and smaller. Thus the cost of high accuracy is weak voltages. In the practical construction of leads, the noise voltages induced at the electrode surfaces and in the body tissues external to the heart might require a definite limit to the maximum number of terms which could be used.

If the heart is a sphere of higher conductivity than its surroundings, it is still possible to produce a unipolar lead type field in the heart. The reason for this is that current distributions on the surface of the body can be found which will produce spherical harmonic fields of any desired order in the sphere.* Indeed, the possibility has already been used in producing a uniform field in a more highly conducting sphere. (The uniform field is the simplest spherical harmonic.) Thus the same procedure just given for a homogeneous body can also be used here. Of course, the actual calculations will be considerably more difficult.

It might be pointed out that the relative intensities of the harmonics are not the same inside the sphere as they are outside, when its conductivity is different from that of its surroundings. In the case of the heart, which has a relatively low resistance, the high order harmonics are attenuated more than the low. This means that the heart tends to make the field within it more uniform than it would be if the heart had the same conductivity as its surroundings.

**Limitations**

There are two different types of limitations that arise in the design of electrocardiographic leads. These are: first, the general limitations that apply to all types of conductors, including the infinite homogeneous conductor, and second, the specific limitations which result from peculiarities in the conductivity of the body that cannot be accurately handled.

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* The proof of this is quite straightforward.
The first limitation arises in the following way. Ideally, it would be desirable to know the intensity and direction of the electromotive forces at every point in the heart, or at least at a given point of interest. Unfortunately, it is not possible to design a lead whose voltage will be proportional to a certain component of the electromotive forces in a limited area of heart muscle. The reason for this is that such a lead would require that the lead field exist only in this small region and be zero elsewhere. If the electrodes of the lead are located on the body surface, they obviously cannot set up such a field.

This situation was first recognized by Helmholtz. He showed that given any distribution of electromotive forces whatever in a conductor, there will always be a distribution of electromotive forces at the surface of the heart which will produce exactly the same external field. Thus the exact nature of the electromotive forces within the heart will always be indeterminate. This does not mean that it is impossible to estimate the approximate location of the electromotive forces. The probability of the lead voltage being due to an electromotive force at a certain point is proportional to the intensity of the lead field at that point. Thus, in the case of the unipolar leads taken with the exploring electrode close to the heart, it is highly probable, although not certain, that lead voltages are produced to a much greater extent by electromotive forces arising in muscle close to the electrode than in more remote regions of the heart.

The second type of limitation, which results from aberrations in the conductivity of the body tissues, comes up when one attempts to extend the procedures for synthesizing leads for a homogeneous body with a more highly conducting spherical or ellipsoidal heart to conductors having more abrupt and more frequent variations in conductivity. What happens, for example, when the heart is not a homogeneous conductor? The authors have studied a number of examples of this situation including, for example, the presence of small spherical or disk-shaped insulators within the heart. In all cases an undesirable change in the field was produced which could not be compensated for. However, large insulators were needed to produce substantial changes in the field.

The clinical electrocardiographer, however, is not interested so much in these theoretical questions as he is in how irregularities in the conductivity of the body and shape of the heart will limit the accuracy of the lead systems constructed according to the methods outlined here. It would be interesting to know how much inaccuracy will inevitably be introduced by the anisotropy of heart muscle, by the small but not negligible difference between the conductivity of heart muscle and blood, by fat pads covering areas of the heart, by a gas bubble at the top of the stomach, and similar factors. Unhappily, there is not enough data available at the moment to answer this question. There is a great deal of experimental work to be done here.

**Summary**

1. This paper, the last of a series of three, explains how to construct leads of any desired type. "Heart vector" and "unipolar" type leads are particularly stressed.

2. An approximate method, involving the selection and combination of leads, is given first. Some simple modifications of the standard leads and the central terminal are worked out, which would increase the accuracy of these leads when interpreted with the concepts of the Einthoven triangle and the central terminal theory of Wilson.

3. An exact method of lead synthesis is advanced next. It is illustrated qualitatively with a simple lead for determining the sagittal component of the heart vector. More involved applications are then considered, which include the construction of "heart vector" and "unipolar" leads that take into account the irregular shape of the body and, to a certain extent, the variations in the conductivity within it.

4. Limitations to the synthesis of electrocardiographic leads are discussed briefly.

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APPENDIX
The Lead Vector of a Tap on a Resistor

In this situation (figs. 9a and 9b) there are three electrodes, \(A\), \(B\) and \(R\). The \(R\) electrode is the reference electrode. The vectors \(\overline{AR}\) and \(\overline{BR}\) are assumed to be known. A uniform resistor, \(r\), with a variable contact or tap is now connected to electrodes \(A\) and \(B\), and one would like to find the lead vector, \(\overline{TR}\), associated with a tap, \(T\), on this resistor and the reference electrode \(R\).

If a source of unit current is connected to this tap, \(T\), and the electrode \(R\), the current will flow through the two parts of the resistor to electrodes \(A\) and \(B\) and then back to the electrode \(R\) through the body. If the tapped resistor is very large* compared to the resistance between \(A\) and \(R\) and \(B\) and \(R\), then the position of the tap will have the controlling influence on how the current divides. To be precise, the current flowing down the part of the resistor connected to the \(A\) electrode will be given by

\[
i_A = \frac{1/(r_{TA})}{1/(r_{TA}) + 1/(r_{TB})} = \frac{r_{TB}}{r_{TA} + r_{TB}} = \frac{r_{TB}}{r} \tag{1}\]

and similarly, the current flowing down the \(B\) part of the resistor will be

\[
i_B = \frac{r_{TA}}{r} \tag{2}\]

A unit current, of course, will flow out from the reference electrode.

The electrode currents described here could have been produced in an entirely different way. Suppose that one had connected a unit source of current to \(AR\), and \(r_{TA}/r\) of a unit source of current to \(BA\). Then the same current will flow through the three electrodes. This means that the lead vector associated with \(TR\) is equal to the sum of the lead vector of \(AR\) and \(r_{TA}/r\) times the lead vector of \(BA\). That is

\[
\overline{TR} = \overline{AR} + \frac{r_{TA}}{r} \overline{BA} \tag{3}\]

This in turn means that the tap will have a lead vector whose tip lies on the line extending from the tip of the \(AR\) lead vector to the tip of the \(BR\) lead vector at a position on the line corresponding

* It is important to realize that this resistance could be as large or larger than the input resistance of an electrocardiograph connected to \(T\) and \(A\), without reducing the accuracy of the instrument. Of course, the electrocardiograph would have to be properly calibrated.

![Figure 9](https://example.com/fig9.png)

Fig. 9. (a) Three electrodes and tapped resistor. (b) Associated lead vectors. See appendix.

exactly to the position of the tap on the resistor. (See fig. 9.)

SUMARIO ESPAÑOL

1. Este trabajo, el último de una serie de tres, explica el cómo construir derivaciones de cualquier tipo deseado. "Vector Cardiaco" y tipo "Unipolar" de derivaciones son particularmente enfatizados.

2. Un método aproximado, sobre la selección y la combinación de derivaciones, se describe primero. Algunas sencillas modificaciones de las derivaciones "standard" y las de terminal central se desarrollan, que podrían aumentar la precisión de estas derivaciones cuando son interpretadas por los conceptos del triángulo de Einthoven y la teoría de Wilson de terminal central.

3. Un método exacto de síntesis de derivaciones se propone después. Se ilustra cualitativamente con una derivación sencilla para la determinación del componente sagital del vector cardíaco. Aplicaciones más complicadas son luego consideradas, que incluyen la construcción del "Vector Cardiaco" y las derivaciones "Unipolares" las cuales toman en consideración la forma irregular del cuerpo y hasta cierto punto, las variaciones en conductividad dentro de este.
4. Las limitaciones en el síntesis de derivaciones electrocardiográficas se discuten brevemente.

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