The Construction of Mean Spatial Vectors From Null Contours

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Experimental and theoretic evidence reveal substantial errors in the determination of mean heart vector orientations exclusively from null contours. Over the limited range of orientations studied, the mean heart vector angle determined by the perpendicular-plane construction departs by as much as 17 degrees from the true angle. These sizable construction errors, strongly dependent upon such factors as heart position and medium shape, are superimposed upon other errors inherent in the many assumptions made in electrocardiographic field analysis.

It would appear self-evident upon examining the history of progress in such fields as biology, biophysics, physical science and others, that methods of electrocardiography which have a firm theoretic foundation based upon known physical laws will ultimately grow to a stature beyond any semiempiric methods. It is in this spirit that a variety of quantitative experiments are being conducted in this laboratory in an attempt to establish a sound experimental and theoretic basis for the analysis of the relationship between the electric currents generated by the heart and the concomitant electric potential differences produced on the surface of the human subject.

In the course of some of these studies it became necessary to develop an accurate technic to measure boundary potentials on the surface of volume conductors with respect to the midpotential of an immersed current dipole. This technic utilizes, in part, a cylindric conductor as a standardizing tank. While this apparatus was designed primarily for torso-model studies, it became obvious that a convenient side pursuit would be the investigation of the validity of the perpendicular-plane construction used in connection with mean spatial vectors and null contours on the human subject. It is the purpose of this paper to present some of the results of this investigation.

Null Contours and Mean Spatial Vectors

Studies of the potential differences between the Wilson central terminal and an exploring electrode placed at a variety of positions on the human subject have led to the definitions and use of three zones on the body surface. A schematic representation of these three zones is illustrated in figure 1 for the case of the QRS complex. The negative zone is defined as that portion of the surface of the human subject where the net area under the QRS complex is negative; the positive zone is defined as the surface where the net area under the QRS complex is positive; the border between these two has been called the transitional zone where the net area under the QRS complex is zero. The same definitions of the three zones are used for the P and T waves, and each wave generally has a different transitional zone. In many human subjects the transitional zones are fairly smoothly shaped bands, approximately elliptical in shape. However, in some cases the transitional zones are highly irregular.

Various methods of analysis utilizing the transitional zones have been proposed. The foundations and assumptions of most of these methods are similar. In essence they amount to plotting the transitional zone as a loop,
CONSTRUCTION OF MEAN SPATIAL VECTORS

Fig. 1. The negative, positive and transitional zones on the surface of a human subject are illustrated at the left for the case of the QRS complex. A homogeneous conducting cylinder representing the human torso is shown at the right in which a mean vector located on the cylinder axis is drawn perpendicular to the plane containing the null contour tilted at an angle $\phi$ with respect to the cylinder axis. The sense of the mean vector has been reversed from the usual convention to clarify this illustration.

called the null contour, on a cylinder of dimensions comparable to those of the human subject as indicated in figure 1. The cylinder is regarded as a rough approximation of the human torso shape, and it is assumed to be a homogeneous volume conductor. A mean manifest heart vector positioned on the axis of the cylinder is postulated to represent the electrical activity of the heart. It is asserted that the orientation of this mean vector is perpendicular to the plane which passes (most closely) through the null contour, the potential of which is assumed to be equal to the mid-potential of the mean vector (dipole). In this way the null contour is associated with the orientations in three dimensions of P, QRS and T mean vectors.* The angles between these vectors are being investigated with the hope of correlating them with heart disorders.

This method of exploring the relationship between the electric activity of the heart and the potentials it produces on the body surface involves many approximations some of which are known to be considerably in error. For example, the Wilson central-terminal voltage, assumed here to remain constant, is believed to vary by sizeable amounts during the cardiac cycle$^{10, 11, 22}$ and this has a very pronounced effect on the location of the transitional zone and, hence, on the mean vector orientation determined from it. Also the representation of the human torso by a cylinder may not be sufficiently accurate for precise work. Furthermore, it has not been established that the human torso can be regarded as a homogeneous conducting medium so far as heart currents are concerned.$^{13}$ In addition, the physical significance of the mean heart vector and its correlation (by some sort of complicated integration) with the more firmly based manifest heart vector which changes its orientation and moment at an irregular rate during the heart beat is obscure. Finally, the perpendicular-plane construction involves considerable error in the determination of the orientation of the mean heart vector. This last defect is the subject of this paper and can be seen to be only one small aspect of the entire problem.

Perpendicular-Plane Construction

It can be seen readily in qualitative terms that the perpendicular-plane construction cannot be correct from a brief examination of a physical system consisting of a current dipole, representing the mean heart vector, located on the axis of a homogeneous conducting cylinder. The dipole produces a current field in the cylinder indicated in figure 2. Since the medium outside the cylinder is a perfect insulator, all the current is confined to the cylinder. This means that the current lines at the wall of the cylinder must be tangent to the cylinder wall. Since isopotentials are everywhere perpendicular to current lines in such a physical system, it becomes obvious

* Additional mean vectors using smaller time intervals also have been defined; for example, the 0.04 second QRS vector.
that all boundary isopotentials must meet the cylinder wall at right angles. The zero-potential surface $V = 0$ which is the dipole midpotential, is no exception to this general physical principle. For distances from the dipole that are very small in comparison with the cylinder radius, the surface $V = 0$ does coincide with a plane perpendicular to and passing through the center of the dipole. (This plane does not pass through the null contour.) However, as the boundary is approached the surface, $V = 0$, must curve away from this plane in order to become perpendicular to the cylinder wall. Thus, it can be seen qualitatively that the perpendicular-plane construction must be in error, and further that the zero-potential surface within the cylinder is not a plane, as has been assumed by others.

The meaning in terms of dipole orientation of this behavior of the zero-potential surface is illustrated in figure 2 which shows a dipole tilted at an angle, $\psi$, with respect to the cylinder axis. The zero-potential surface, which is S-shaped in cross section, associated with the dipole establishes the null contour on the cylinder wall. The perpendicular-plane construction, indicated by the dashed line through the null contour, leads to an apparent tilt angle, $\psi'$ (of the dashed vector drawn perpendicular to the plane through the null contour), which is smaller than the true tilt angle of the dipole that produced this null contour. The discrepancy between the true and apparent orientations is seen to be very sizeable in figure 2 which is drawn approximately to scale. Quantitative measurements of the actual size of the errors entailed in this construction have been made with the apparatus and method described below.

**Experimental Method**

The location of the zero-potential contour on the wall of a homogeneous conducting cylinder as produced by a finite dipole* was studied experimentally for both centric and eccentric dipoles. The basic method used is described elsewhere along with some of the design problems.† A description of the experimental arrangement can be given with reference to figure 3. The finite dipole, consisting of two $3\frac{1}{2}$ inch-radius nickel discs separated by a small $3\frac{1}{6}$ inch insulating spacer, is immersed in a tap-water† filled plexiglass cylinder of 7 inches radius and 30 inches length by means of an insulated water containing current leads. The dipole can be positioned (by the assembly shown which is equipped with various scales and dials) in depth of immersion and distance from the cylinder axis. The dipole orientation is adjusted by a pivot at the bottom of the supporting rod, and also by twisting the rod about its axis. A potential difference is applied to the dipole through a resistance-capacitance bridge and the dipole current $I$ is distributed throughout the cylinder. The bridge detector, connected between watertight pick-up electrodes on the cylinder wall and the junction between the bridge resistors $R_1$ and $R_2$, consists of a 40-decibel preamplifier in cascade with a harmonic wave analyzer tuned to the operating frequency of 1000 cycles per second.‡ Complete shielding, not shown in figure 3, is employed throughout.

The ground side of the detector is established at

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* A finite dipole is defined as a current source and sink separated by a finite distance. It differs from a mathematical dipole in that the latter source-sink pair has infinitesimal separation with a finite product of source-strength times separation. In practice a mathematical dipole can be approximated closely by employing current-electrode spacing and size that are small compared with other dimensions of the system.

† The zero-potential contour is independent of the resistivity of the homogeneous medium. Tap water was used for convenience.

‡ The potential distribution at 1000 cycles per second is negligibly different from that at lower frequencies. This frequency was selected as a practical compromise among adverse effects of electrode polarization, system-shielding and obtainable bridge balance. There is no measurable phase shift at this frequency.
the desired zero reference potential, which is the electrical midpoint between the two current electrodes taking the discontinuity owing to polarization into account, by first optically aligning the finite dipole on the cylinder axis midway between the end caps. The detector is then connected to a boundary pickup electrode which lies on a circle whose plane passes through the geometric midpoint of the dipole. Then the bridge is balanced by adjusting $R_1$ and $C_1$ for a null reading on the detector. This procedure circumvents the pronounced adverse effects of electrode polarization.

With a dipole current of 0.4 milliampere at 1000 cycles per second, the applied voltage to the dipole is 0.05 volt and the bridge null remains less than 0.5 microvolt for short periods (20 to 30 minutes) and less than 5 microvolts for periods of four to five hours. These null voltages correspond to bridge balances of 100,000:1 (100 decibels) and 10,000:1 (80 decibels), respectively. The small departures from zero are attributable to thermal gradients in the medium, polarization-impedance fluctuations and irreducible effects of stray capacitance and stray pickup voltage. The maximum boundary voltage on the cylinder under the above conditions is 0.6 millivolt; thus, it can be seen that even long-period drifts of 5 microvolts represent an error in the reference potential of less than 1 per cent of the maximum boundary voltage.

With the reference potential established in this manner, the detector can then be connected to other pickup electrodes and the dipole position can also be changed. The detector voltage readings (which are unbalanced bridge voltages) represent voltages with respect to the dipole mid-potential since the potential established at the grounded point of the bridge remains as initially set regardless of shifts of dipole position or connections of the detector to various pickup electrodes. Therefore, the null contour on the cylinder boundary can be measured directly* for a variety of dipole positions and orientations.

**RESULTS**

Using the method described, the null contours on the wall of the cylinder were determined for various known positions and orientations of the finite dipole for both centric and eccentric cases. The complete results have been published elsewhere. A portion of the centric results will be given here.

For centric dipoles located midway between the endcaps of the cylinder and tilted with respect to the cylinder axis by angles ranging from 0 degrees to 45 degrees (or 135 degrees to 180 degrees) the null contour is approximately an ellipse (to an accuracy of a few per cent for tilt angles equal to 20 degrees or less) and is indicated by the shaded band in figure 4. In order to describe the null contour it is convenient to designate the $z$ coordinate of the null contour with respect to a plane perpendicular to the cylinder axis which passes through the dipole, defined by $z = 0$. The line joining the points where the null contour passes through $z = 0$ is perpendicular to the plane in which the dipole is tilted, while the maximum values of $z$, designated by $z_m$ in figure 4, occur symmetrically in the plane in which the dipole is tilted.

It can be shown that the null contour is a

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* Usually the null contour lies between pairs of pickup electrodes which are fixed in position, and must be computed by interpolation.
The equation \( \tan \psi = \frac{\pm \int_0^\infty \sin \left( \frac{kz_m}{a} \right) dk}{\int_0^\infty k \cos \left( \frac{kz_m}{a} \right) dk} \)

where \( \psi \) is the tilt angle between the dipole axis and the cylinder axis, \( I_0(k) \) and \( I_1(k) \) are modified Bessel functions and \( k \) is a variable of integration.

Fig. 4. A centric dipole tilted at an angle \( \psi \) with respect to the cylinder axis is shown at the right with the approximately elliptical null contour it produces, indicated by the shaded band. The experimental results are shown at the left and compared with a theoretical curve which pertains to an infinitely long cylinder, and which lies slightly above the experimental curve as expected.

The case of a centric dipole, the erroneous tilt angle, \( \psi' \), (see fig. 2) is related to the maximum coordinate of the null contour by the equation \( z_m/a = \tan \psi' \), which enables \( \psi' \) to be calculated if \( z_m/a \) is known. The erroneous tilt angle is always less than the true tilt angle owing to the bending over of the zero-potential surface within the cylinder as illustrated in fig. 2.

For a numerical example, suppose \( z_m/a \) is assigned the values \( \pm 0.33 \), which corresponds to a true tilt angle of 30 degrees as can be seen in fig. 4. The erroneous tilt angle is then given by \( \psi' = \tan^{-1} 0.33 = 18.3 \) degrees, a discrepancy of about 12 degrees in this case.

A plot of the erroneous tilt angle obtained by means of the perpendicular-plane construction versus the true tilt angle is given in fig. 5. This curve was derived by the method of calculation illustrated in the above example. The curve is very nearly a straight line with an average slope of approximately 0.6, which means that the angle obtained from the perpendicular-plane construction is about 60 per cent of the actual tilt angle.

When the dipole is moved to an eccentric position, the simple and symmetric behavior exhibited by the centric dipole field is no longer observed. The discrepancy between the true angle of dipole tilt and the apparent angle, as measured by null contour technique, cannot be described by a simple constant correction factor as in the centric case, but assumes major complexities. With an eccentric dipole the null contour deviates markedly from an ellipse,
and the constructed plane which fits this contour most closely does not contain the dipole center. Furthermore, deviations between the true tilt angles and apparent angles obtained on the arbitrary assumption of a centric dipole, exceed the 60 per cent discrepancy applicable to the centric case. The influence of eccentricity is presented in detail elsewhere.\textsuperscript{14}

**Discussion**

It will be noted that dipole tilt angles up to only 45 degrees were employed. This was done mainly because the cylinder end caps play an increasingly important role for larger tilt angles; indeed, a portion of the null contour appears on the end caps rather than lying entirely on the cylinder wall for tilt angles that are too much in excess of 45 degrees for the cylinder used experimentally. The results of a study of the end-cap effects would apply only for the specific cylinder used and would not be as general as those presented. Although the correction factor for tilt angles exceeding 45 degrees has not been explored, it is obvious that the factor 1.6 is not applicable for all tilt angles because when this angle is 90 degrees, for instance, the null contour and perpendicular plane can be seen to agree exactly from symmetry.

The theoretic complexities of the cylindric volume conductor are well illustrated by the null contour equation for an infinitely long cylinder which appears in integral form and involves nonelementary functions. The solution for a cylinder with end caps is even more complicated; so much so that it becomes rather impractical to utilize the solution conveniently. Furthermore, the null contour behavior when the dipole position is not on the axis of the cylinder lends a considerable amount of additional complication.\textsuperscript{14}

The dipole angle error entailed in the perpendicular-plane construction depends, in part, on the shape of the homogeneous volume conductor in which the dipole is immersed. This study has been devoted to cylindric conductors because they have been advocated in the application of the null-contour technic. However, when the perpendicular-plane construction is applied to both centric and eccentric dipoles in a homogeneous spherical volume conductor, the errors of the construction are surprisingly small, as can be shown by unpublished theoretic analyses. For dipole eccentricities up to 20 per cent of the sphere radius, the maximum discrepancy between the true angle and the apparent angle obtained from the perpendicular-plane construction is only approximately 3 degrees, and is less than 6 degrees for dipole eccentricities as large as 40 per cent of the sphere radius. These errors are much less than those encountered in a cylindric conductor. The claim has been made that the cylindric representation of the human subject is superior to the spherical representation; however, from the standpoint of the perpendicular-plane construction itself, the

![Graph](image_url)
use of a spherical medium has the advantage of negligible constructional errors.

In these studies it is shown that the analysis of vectors solely by the null-contour technic in a cylindric system results in large errors inherent in the utilization of the perpendicular-plane construction itself. Even in the most simplified case, with centric position of the dipole at a tilt of not greater than 45 degrees from the vertical, the true angle of tilt is greater than the constructed angle by approximately 60 per cent. Since, within the restrictions listed in figure 5, the disparity between the true and apparent angles increases linearly with increasing tilt, it is possible in a simple manner to convert apparent angles to the true angles in the idealized schema. The application of a correction factor might be attempted in the human subject according to the following example. Suppose the total difference between the maximum z-coordinates of the null contour is \( A \) inches, and the cylinder fitted to the human subject has a diameter of \( B \) inches. The dipole tilt angle given by the perpendicular-plane construction is then \( \psi' = \tan^{-1} \left( \frac{B}{A} \right) \). This angle may then be multiplied by 1.6 to obtain a corrected result. For instance, if \( A = 5 \) inches and \( B = 12 \) inches, then \( \psi' = \tan^{-1} \left( \frac{5}{12} \right) = 22.6 \) degrees, which is the angle computed from the perpendicular-plane construction. The actual dipole tilt angle is then given quite closely by 1.6 (22.6 degrees) \( \approx 36 \) degrees. The application of this factor of 1.6 to the human subject is necessarily a first approximation at best, and of very dubious value because of the obviously eccentric position of the human equivalent dipole, if for no other reason.

An alternative method for determining the mean vector spatial orientation is also used. Precordial leads are used to establish the point at which the null contour intersects with the transverse plane through the heart center. Limb-lead data provide the requisite additional information. Errors entailed in this method are quite different from those relying exclusively on the null contour. Therefore, the results of the two methods cannot be expected to be in agreement.

Whereas it is generally conceded that vector analysis of the electrical field of the heart by the null contour technic yields only approximate results, it is shown here that the errors resulting from misapplication of physical principles are of considerable magnitude. These errors are superimposed upon the limitations inherent in the many assumptions applied to field analysis of the electrical forces of the heart.

**Summary**

1. The essential ideas underlying mean spatial vectors and null contours are reviewed briefly. The perpendicular-plane construction in a cylindric volume conductor, as employed in conjunction with null contour for the measurement of heart vectors, is clarified conceptually.

2. Experimental apparatus for the quantitative determination of true dipole angles in such a construction is described.

3. The discrepancy between true dipole angle and apparent angle obtained from the null-contour, perpendicular-plane construction for the most simplified case, a centric dipole position, is found to bear a ratio 1.6/1.0 with the restriction of a maximum 45 degree tilt of the dipole from the vertical.

4. The dependence of the dipole angle error on boundary shape is discussed.

5. Limitation of the application of a "correction factor," effective in the idealized cylindric schema, to the null-contour technic is emphasized.

6. Major limitation of the usefulness of the null-contour technic for analysis of spatial vectors on other than an empiric basis is implied.

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**Sumario Español**

1. Las ideas esenciales sustentando los vectores espaciales promedio de los contornos tornados se repasan brevemente. La construcción de plano perpendicular en un volumen cilíndrico conductor, empleándose en conjunción a con-
tornos tornados para la determinación de vectores cardíacos, se clarifica conceptualmente.

2. Aparatos experimentales para la determinación cuantitativa de ángulos dipolos verdaderos en tal construcción se describe.

3. La discrepancia entre ángulos dipolos verdaderos y ángulos aparentes obtenidos de los contornos tornados de construcción de plano perpendicular en el caso mas sencillo, una posición dipolo céntrica, se encontró tener una razón de 1.6/1.0 con la restricción de una inclinación máxima de 45° del dipolo con la vertical.

4. La dependencia del error de ángulo dipolo en la configuración del lindero se discute.

5. La limitación de la aplicación de un “factor de corrección,” efectivo en el esquema cilíndrico idealizado, al uso clínico de la técnica de contorno tornado se enfatiza para el estimado absoluto de dirección vectorial y para la comparación de los ángulos QRS-T.

6. Mayor limitación del uso de la técnica de contorno tornado en el análisis de vectores espaciales en otra que en una base empírica se implica.

REFERENCES

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