Electrocardiographic Leads

II. Analysis

By Richard McFee, M.S., and Franklin D. Johnston, M.D.

In this second paper of a series of three on electrocardiographic leads, presented primarily from the standpoint of an electrical engineer or physicist, the concept of the lead field introduced in the first paper is considered in detail and several necessarily indirect methods for the estimation of these fields are presented.

In the first paper of this series\(^1\) it was shown that the relationship between the electromotive forces of the heart and the voltages produced by them in a given lead are completely determined by the electric field set up in the heart when a unit current is introduced into the lead. This means that to analyze the relationship between the electromotive forces and the lead voltage it is only necessary to determine the nature of the lead field. A brief discussion of various methods of studying these fields is given in this paper.

When a current is introduced into a lead the resultant flow of current through the heart produces potential differences between various points in the muscle. If it were possible to measure these potential differences, this data would furnish the desired information about the field. Unfortunately, it is not possible to do this with human subjects. For this reason it is necessary to resort to other approaches, all essentially indirect.

The simpler of these approaches furnish graphic patterns in which one component of the lead field current is taken as zero. Such two-dimensional patterns are easy to obtain and to interpret, and frequently give an insight into the nature of the lead which would be quite difficult to obtain from three dimensional data, since the latter cannot be conveniently represented graphically. However, unless one component of the actual lead field is small

\(^{1}\) From the Department of Internal Medicine, University of Michigan Medical School, Ann Arbor, Mich.

This study was made possible by a grant from the S. S. Kresge Foundation to Dr. Frank N. Wilson.

(as is probably the case with the standard leads) the patterns obtained from such studies cannot be used for quantitative purposes, such as estimating the accuracy of a certain interpretation of a lead. To obtain quantitative information, three dimensional studies are usually necessary. These are much more difficult than the two-dimensional ones, not only because of the extra dimension and the greater accuracy usually sought for, but also because of the difficulty in representing the data about the field in a fashion easily grasped.

In this paper both two- and three-dimensional methods for studying the fields are considered, but only the basic principle and the highlights of each method are given. For the convenience of the average reader, nearly all mathematical considerations have been placed in the appendices. Some of the important problems associated with the analysis of leads, such as the "validity" of the Einthoven triangle and Wilson central terminal, are touched upon in the discussion.

Two-Dimensional Approaches

Sketching the Fields. A surprising amount of information about lead fields can be obtained from simple sketches of the flow lines, based upon intuition. The ability to do this naturally improves with practice. It is worth while to keep in mind the analogy between the flow of water (without turbulence or eddies) and the flow of electricity. To the extent that the body is electrically homogeneous, it may be thought of as a hollow container into which water rather than electricity is entering or leaving the body at the lead electrodes. To
illustrate the idea involved here, consider lead I. If the body were hollow and if water were to enter the body at the right arm and leave from the left, the flow lines would be more or less like those shown in figure 1. From the symmetry of the body it is clear that, if the heart were located in the center of the chest, the direction of the flow lines within it would, on the average, be transverse, in general agreement with Einthoven's interpretation of lead I. However, since the heart is usually located a little to the left of center, the flow lines within it will tend to point slightly upward, and this in turn means that lead I is probably influenced slightly by the vertical component of the heart vector.

It is quite obvious, even from rough sketches of the lead fields involved, that most leads used at present for obtaining the sagittal component of the heart vector are far from satisfactory. For example, consider the lead proposed by Duchosal and Sulzer.\(^2\) A sketch of the field of this lead is shown in figure 2A, and should be compared with the field desired, which is sketched in figure 2B. The field of the Duchosal's lead suffers most from having the wrong average direction, and it is possible that the lead, as a result, is as sensitive to the vertical component of the heart vector as it is to the sagittal component.\(^*\)

The field associated with the lead formed by the back electrode of the tetrahedron\(^3\) and a second electrode connected via equal resistances to the foot and neck is sketched in figure 2C. (This second electrode is the two-dimensional equivalent of the central terminal.) Although this field has the correct average direction, it is by no means uniform. Evidently the lead is much more sensitive to electromotive forces on the back of the heart than to those in front; that is, the lead acts more like a proximal or chest lead than like a heart vector lead.

* Where the three-dimensional situation is considered, a transverse component is probably involved also.
It is not difficult to see from the ideal sagittal lead pattern shown in figure 2B how a sagittal lead must be constructed. This topic will be treated in the next paper, but in the meantime the curious reader may find it interesting and illuminating to try to work out this lead for himself.

Fluid Mappers. The hydraulic analogy mentioned previously in connection with sketches of the field is the basic principle of simple devices for studying the flow patterns, which are called “fluid mappers.” Their application to electrocardiographic problems has been described in detail in a recent paper by McFee, Stow and Johnston. In fluid mappers the water flows in sheets between a glass plate and a flat plaster slab, past small crystals of soluble dye, which dissolve slowly, thus making the lines of flow visible. The effective resistance to the flow of water, which corresponds to the electrical resistance of the tissues of the body, is easily varied by changing the depth of the “flow space;” that is, by altering the thickness of the fluid sheet.

Figure 3 shows a typical fluid mapper pattern. It represents the field of a lead formed by an exploring electrode on the precordium and a second electrode representing the central terminal, which is connected to the foot and neck through equal resistances. The depth of the flow space in figure 3A is constant, while in figure 2B it is so altered that the resistance of the area representing the heart is one-fourth that of the other regions of the mapper. These mappers were built in order to find to what extent the central terminal is “indifferent” when the exploring electrode is over the heart. As was pointed out in the first paper of this series, if the central terminal were “indifferent,” the flow lines within the heart would appear to radiate from the exploring electrode in straight lines. Inspection of both patterns indicates that this is not far from the case.

Fluid mapper patterns furnish no direct information concerning the intensity of the field at various points. However, this information can be obtained by careful study of the

FIG. 3. Fluid mapper patterns of central terminal chest leads. (A) Body has uniform conductivity; (B) heart has twice conductivity of rest of body.

flow lines and isopotential lines in a manner which is described in appendix I.

Conducting Paper Mannikins.* Two-dimensional lead fields in homogeneous media may be studied in a simple fashion by using conducting paper.† This paper comes coated with a thin even layer of a material having a moderate electrical resistance. The paper is cut to the desired outline, and tacks are applied to the sites of the electrodes of the lead. The electrodes are then connected via resistances (30,000 to 300,000 ohms) to a battery (45 to 450 volts) and the resulting field in the paper established with two needle-tipped probes and a sensitive vacuum tube voltmeter (either 1 or 3 volts, full scale, input resistance around 10 million ohms). The voltage measured will be zero if the two electrodes are on the same isopotential, and this fact enables one to locate quickly many such points. After this has been done the isopotential lines can be drawn with the help of a French curve. Figure 4 shows an isopotential pattern of a special chest lead which has been obtained using this method. This lead, which is called a “null” lead, will be discussed shortly.

Fields on the Chest Surface. It is relatively

* Since the writing of this paper the authors have learned that a similar use of these mannikins has been made by Brody and Romans.12
† “Teledeltos” paper. The Western Union Company, New York City.
ELECTROCARDIOGRAPHIC LEADS. II

Fig. 4. Isopotential pattern of "null lead" obtained from conducting paper mannikin.

easy to measure the lead field produced on the chest surface over the heart, and there is no question that this field reflects (to an unknown extent, unfortunately) the field within the heart itself. Such measurements can be made either by determining the potential differences produced on the chest when a current is introduced into the lead, or by measuring the voltage produced in the lead when a current is introduced into the same pair of electrodes on the chest. The reciprocity theorem insures that the same reading will result no matter which procedure is followed. Experiments of the second kind have been described by Wilson, Bryant and Johnston, and experiments of the first kind have been performed, but apparently not reported, by Schmitt and Levine.

Some of the data obtained by Wilson and his associates is shown in figure 5. The arrows in this figure represent the measured intensity and direction of the surface potential gradient at the central points of the arrows. In figure 5 the flow lines (dashed lines) of the field have been sketched with the intensity and direction of the arrows in mind. It is assumed here that the resistance of the chest surface is the same regardless of the direction of the current flow (isotropic). If this were not so, the direction of the current flow lines would deviate somewhat, in general, from the direction of the potential gradient.

The central terminal leads have also been studied by investigating the fields produced on the chest surface. A series of unpublished experiments were performed by one of the authors (R. M.) in which the fall in potential about the exploring electrode was measured when it and the central terminal were connected to a source of current. These experiments were difficult to interpret because the potential did not drop off completely uniformly on all sides of the exploring electrode, but rather exhibited irregularities of as much as 25 per cent. These were apparently due to variations of conductivity produced by the underlying ribs. When measurements on four sides of the electrode were averaged, they
Three-Dimensional Approaches

Mathematical Models. No matter how complex the shape of the body or the variations in its conductivity, it is possible, in principle, to find the field of a lead by mathematical methods. In most cases, however, it is much easier to construct an electrolytic model and make measurements on it than it is to attempt such an analysis. It is only when one is interested in relatively general aspects of the field that the use of mathematical models is justified.

An example of a situation of this sort was given in the first paper of this series, where the infinite homogeneous conductor was used as a model for studying the accuracy of interpreting the central terminal chest leads as "heart vector" leads. Its use might be justified by the fact that the difference between the ideal field associated with the interpretation and the actual field is probably considerably greater than the difference between the actual field and the field in the model, at least when the exploring electrode is close to the heart.

The mathematical approach is also useful in studying the over-all characteristics of leads. For example, the interesting properties of the rather unusual leads shown in figure 6 ("null" leads) may be investigated by assuming that the part of the body underneath the plane of the chest surface extends to infinity and is, in addition, homogeneous. This assumption leads to the following analysis.

When a source of current is connected to the lead shown in figure 6A, the current streams into the conductor through electrodes A and C, and out through electrodes B and D. Along the line perpendicular to the plane of the electrodes, which passes through the center of the square formed by them, the net current is zero, since the electrodes are so arranged that their contributions to the current flowing along or across this line cancel each other. The result of this is that the lead is insensitive to electromotive forces existing at points along this line.

More or less the same thing happens when the lead shown in figure 6B is used, except that in this case the null occurs at only one point on the center line. As shown in appendix II, the distance \( d \) of this point from the surface of the conductor is related to the ratio of the resistances \( R_s \) and \( R_t \), and the distance \( r \) from the center to the outside electrodes by the equation

\[
\frac{R_c}{R_t} = \left(1 + \frac{r^2}{\partial^2}ight)^{1/2} - 1 \quad (1)
\]

This equation can be used to adjust the ratio of \( R_s \) to \( R_t \) so that it will give a null in the field at a certain depth and thus be insensitive to electromotive forces at that point. The two-dimensional approximation to the field of this lead is shown in figure 4.

A practical, clinical version of the "null line" and "null point" leads just described can be constructed by attaching five electrodes to a single frame which can be freely moved about on the surface of the chest, and by using the central terminal as an indifferent electrode. The extent to which the actual fields would differ from the ideal fields in the semi-infinite homogeneous conductor depends upon many factors, such as the spacing of the electrodes, the effects of the ribs and subcutaneous fat, the relative resistance of the blood, heart
Electrocardiographic leads. II

Muscle and lung. However, it seems quite likely that in general the nulls of the leads would occur more or less at the locations predicted in the preceding analysis. An electrode arrangement of this sort might prove useful in the location of recent infarcts. One would start with the "null line" arrangement (fig. 6A) and move the electrode frame about until a position for it is found that eliminates the QRS, RS-T segment or T-wave change in the electrocardiogram due to the infarct. (At this position rotation of the frame about the center should produce no change in the part of the ventricular complex under investigation.) Next, one would change to the "null point" arrangement (fig. 6B) and adjust the ratio of $R_c$ to $R_l$ until the abnormality due to the infarct again disappears. If the infarct is small, and the field of the lead reasonably like its theoretic model, the infarct should then be located directly beneath the center of the electrode frame at a depth corresponding to the ratio of $R_l$ and $R_c$ that yields a "null."*

Another example of the use of mathematical models in studying lead fields is the analysis of the homogeneous sphere, which has been reported recently by Wilson and Bayley.7 It is interesting to note that had Wilson and Bayley used the concept of the lead field in their analysis, rather than attempting to find the field of a dipole, they would have obtained the desired result with considerably less effort.

Cadavers, Phantoms and Catheters. Although it is not possible to measure potential differences produced in the heart tissue of a normal human subject, such experiments can be performed with a cadaver, or with an artificial electrolytic model of the body, such as the "phantom" of Burger and van Milaan.8 Furthermore, as Dr. Frank Wilson suggested to the authors several years ago, it is possible to get a general idea of the field produced in the ventricular cavity of a normal subject by using a catheter equipped with suitable electrodes. These three methods all require a similar type of electrical measurement, and before discussing their advantages and disadvantages a few words about these measurements are desirable.

The general procedure involved is this. A source of current is connected to the lead and the voltage produced between a pair of electrodes on or in the body is measured. Although this procedure sounds simple, experience has shown that enormous errors can result from almost imperceptible mistakes in technic.† One simple procedure which avoids many such pitfalls is the following. A high voltage battery (45 to 180 volts) is used as a current source. It is put in series with a high resistance (50,000 ohms or more) which maintains the current constant, regardless of polarization or variation of contact resistance at the electrodes. (This method of obtaining constant current is well known; see, for example, reference 9.)

The current is monitored with an accurate milliammeter and turned off and on with a telegraph key. To insure that there is no leakage of current over undesirable paths, the battery is placed on a dry insulated platform, and the key operated only by the insulated knob. An electrocardiograph employing a mirror galvanometer, preferably battery powered, is used to measure voltage. Its record permits accurate measurement of the deflections and the high-input impedance minimizes the effect of high-contact resistance and polarization on the part of the electrodes connected to the machine. Battery power enables the electrocardiograph to be operated without a ground, thus avoiding the consequences of improper "differential" action on the part of the amplifiers.‡ Records of the calibration voltage of the electrocardiograph are obtained with each measurement of the voltage produced by the lead field.

The pickup electrodes employed should be

---

* It is theoretically possible for there to be distributions of electromotive forces within the heart which do not set up localized "injury potentials" yet which nevertheless appear as infarcts on the electrocardiographic record. However, such distributions are physiologically improbable, if not impossible.

† The two main sources of trouble are (1) sensitivity of the amplifiers to the "common mode" as well as the "differential" component of the input signal and (2) flow of current through the "ground" electrode.

‡ If the electrocardiograph is line powered, the ground should be put near the pickup electrodes connected to its input.
as small as possible, and their spacing be many times their maximum diameter. If they were large and closely spaced, their effective separation and relative orientation might differ considerably from that expected as a result of variations in contact resistance and polarization of the electrode surfaces.

Two tests should be made before every set of measurements. First, the two wires of the current source are connected to one terminal of the lead and the switch (telegraph key) turned on and off. The electrocardiograph beam should not be deflected. Next, the current source is reconnected to both of its electrodes, and the two wires of the electrocardiograph are attached to one of the pickup electrodes. Again, no deflection should be produced when the source is switched on and off. If there is a deflection in either case it is definite proof that the electrical setup is improper, and that there will be errors in the measurements.

Let us now consider some of the merits of the methods of studying lead fields mentioned earlier in this section.

The study of fields in cadavers would be the perfect solution to the problem of determining the fields if it were not for the changes in the resistance of the tissues that occur when circulation ceases, and for the difficulty in inserting the electrodes into the heart muscle without producing significant changes in the resistivity; for example, by the introduction of air in the opening or puncture of the chest and heart tissues.

Numerous experiments with cadavers have been made, the most extensive of which were performed recently by W. denBoer. In these experiments the lead vectors were found by forcing electrodes through the chest wall and measuring the voltages produced in various leads when an alternating voltage was connected through a fairly small resistor to the two electrodes. Considerable data was obtained showing variation of lead vector with location of the electrodes, and from one cadaver to another.

The study of the fields in electrolytic models is considerably easier than experiments involving cadavers, because of the greater ease and reliability of the measurements involved. However, models of this sort may always be criticised (with or without good cause) on the grounds that they fail to mirror exactly the detailed fluctuations of the resistance of the body.

A very simple electrolytic model of the body can be made with a box of some insulating substance such as plastic or glass, and this has been done by Bryant. The length of the tank should be about twice its width. A glass goldfish aquarium with these relative dimensions has been used by the authors. More complex models can be made like those of Burger and van Milaan who, in addition to giving their model lifelike shape, also introduced variations of conductivity by representing the lungs and liver with bags of sand. In the use of electrolytic models, the smaller the electrodes and their spacing, naturally the more detailed the data one can obtain about the field. This data can be most conveniently represented in a simple table of the measurements of the potential gradients in three reference directions at various points in the “heart.”

Enough measurements should be made to permit accurate interpolations of the values of the components at intermediate points.

Perhaps the most exact method for studying lead fields is by intracardiac leads through the use of catheters. In this case it is certain that the resistivity of the various organs and the shape of the body are correct. However, experiments of this sort are the most difficult to make and to interpret, particularly when a detailed picture of the field is desired. In addition, the measurements concern the field in the cavities of the heart, rather than in the muscle itself, where the electromotive forces are generated.

One experiment of this sort has been reported in the literature. Here, the current was introduced into the two electrodes on the tip of the catheter, rather than into the lead, a technic which may involve the risk of ventricular fibrillation, and should be cautiously used for this reason.

The following procedure would seem preferable. A catheter is obtained which has one electrode on its tip and another three to five centimeters behind it. This catheter is inserted
into one of the ventricles and the position of the two electrodes found by means of x-ray films in the frontal and sagittal planes. (The spacing of the electrodes determined from x-ray pictures must be corrected for the flare of the x-ray beam.) Then a current of from 1 to 5 milliamperes is introduced into each of the leads being investigated and the resultant potential differences between the catheter electrodes measured with a vacuum tube electrocardiograph. Next, the procedure is repeated, with the catheter electrodes in a position as nearly as possible perpendicular to their axis in the first case. Finally, the procedure is again repeated, with the catheter electrodes in a position nearly perpendicular to the plane of the first two axes. The deflections measured this way, when properly calibrated and inserted into the formulas given in appendix III, will yield the three components of the average potential gradient in the cavity where the catheter was located. The lead field current is determined by finding the specific conductivity of the blood and multiplying the potential gradient by it.

An alternate procedure requires the construction of a catheter with four electrodes, so spaced that when the end of the catheter is curled up in the cavity the fourth electrode completes one turn.* If this is done, only one set of x-ray photographs is required, and the catheter need not be moved after it is first put into position. The formulas in appendix III may also be used in this case.

A more detailed picture of the field in the cavity can be obtained with a catheter having more than four electrodes.

In concluding this section on the use of cadavers, phantoms and catheters, it might be pointed out that the various technics can be combined. For example, by putting the excised heart of a cadaver into an electrolytic tank one can obtain more easily than with an entire cadaver a detailed picture of the field within the heart muscle itself, particularly since the resistivity of the latter probably does not change a great deal with failure of circulation. Again, the field within the ventricular cavity of a cadaver can be studied, using the catheter technics. Or again, the procedure discussed earlier for mapping the potential differences on the chest surface might be applied to both cadavers and normal subjects with the object of showing up in this way some differences in the resistivity of cadavers and normal subjects.†

**DISCUSSION**

None of the different methods for the study of leads and their fields outlined above have been extensively used, but alone or in combination they are powerful tools for the investigation of electrocardiographic leads and much has been learned from them.

For example, evidence suggesting that the Einthoven triangle concept is sufficiently accurate for clinical purposes has come from the work by Burger and van Milaan in their "phantom," from the catheter studies of Butterworth and Thorpe, and from the work employing fluid mappers reported by McFee and associates. The authors do not believe that the vectors calculated by the use of the Einthoven triangle are exact but think they are fairly close to the true vectors. Some additional evidence supporting this view is provided by calculations carried out from patterns obtained with fluid mappers according to the general method described in the first paper of this series.

These calculations indicate that the Einthoven triangle method is approximately 77 per cent accurate. Readers who are interested in the details of these computations may get them from one of the authors (R. M.).

Although most cardiologists now believe that the central terminal is the best "indifferent" electrode currently available, this view is not held by all. Furthermore, there is little information concerning the situations or conditions under which the central terminal may provide an excellent indifferent electrode and those where it may not be entirely satisfactory.

---

* The end should be curled up in a helix, such as is formed by the thread of a screw. The distance between turns (pitch) should be somewhat greater than half the diameter of the helix.

† This procedure can also be used to test electrolytic models.
Study of lead fields that exist when the central terminal is employed have already been informative. Fluid mappers have been particularly useful. An analysis of the accuracy of the central terminal arrangement was carried out on the fluid mapper pattern shown in figure 3A of this paper. This was done by the method outlined in appendix I and indicated an accuracy of 92.5 per cent for this particular lead.

Although the Einthoven triangle scheme may be used to determine the components of the heart vector in the frontal plane with reasonable accuracy, all leads commonly used at the moment to obtain the sagittal component of the heart vector appear to be unsatisfactory. (See fig. 2 and associated discussion.) Our studies indicate that there is only one type of lead which can be used for this purpose. It will be described in the third paper of this series.

In recent years many investigators have attempted to find whether or not different leads were alike by comparing records of the voltages produced in them by the heart. This method of studying leads may sometimes prove to be very unreliable. The electrical forces of the heart may be such that voltages in leads with dissimilar fields turn out to be alike. For example, if the direction of the heart vector were to remain vertical throughout the beat, then the voltages in leads I and II would be alike, even though the leads themselves are quite different. For this reason, we feel that definite conclusions about the nature of leads should not be based solely on studies comparing voltages in different leads.

**Summary**

This paper, the second of three which deal with the study of electrocardiographic leads primarily from the electrical point of view, contains a brief discussion of a number of experimental and theoretic technics which may be used for determining the field of a given lead. The importance of such fields has been pointed out in the first paper of this series.

Methods to obtain patterns representing two-dimensional approximations of the fields are considered first. It is shown how simple sketches, based upon one's intuitive understanding of the field, will quickly reveal the general character of the lead. Such sketches are used to point out the shortcomings of several leads which have been used to obtain the sagittal component of the heart vector. Special devices for studying two-dimensional fields, such as "fluid mappers," are then discussed briefly.

Methods of obtaining information about the actual three-dimensional fields are considered next. The use of simple mathematical models is illustrated by the analysis of a new type of lead which is insensitive to electromotive forces at certain locations. The procedure for calculating the general direction of the field from measurements of the voltage produced between the electrodes of a catheter is described in detail.

**APPENDIX I**

*Quantitative Calculations from Flow-Line Patterns*

The relative intensities of the field at different points in a two-dimensional flow-line pattern can be determined by measuring the spacing of the lines, since this spacing varies inversely with the intensity of the current flowing between the lines; that is, if the spacing is halved the current intensity is doubled. This is due to the fact that the current flowing between any two lines remains constant, regardless of the spacing.

If a line going from one part of the field to an adjacent point is perpendicular to the direction of the current flowing between them, then there will be no potential difference between the points, since there is no component of current to create a voltage drop in that direction. If a whole series of points are connected to adjacent points by lines perpendicular to the current flowing between them, the line that is formed is called an "isopotential." When any flow line and isopotential cross, they are always at right angles to each other. The isopotentials may be used in place of the flow lines to determine the direction of the current flow, since it is known that the latter are always perpendicular to the isopotential lines. The spacing between adjacent isopotentials also varies inversely with the intensity of the current passing from one to the other, provided that the resistance of the medium is constant. The reason for this is that closely spaced lines indicate a rapid change in voltage, and this, by Ohm's law, is accompanied by a correspondingly intense current.

Consider now the determination of the relative intensity of the current at two different points in the heart due to the field of the same lead. One begins with a flow-line pattern of the lead under considera-
Fig. 7. (A) Sketch demonstrating quantitative analysis of a flow line pattern. (B) Sketch showing that if the directions of the lead vectors of the standard leads are known, then their relative magnitudes can also be determined.

The “Null Point” Lead

Although this lead actually consists of five electrodes, its symmetry is such that the null may be calculated from the arrangement shown in figure 8. Here the current enters the chest surface via electrodes A and B and leaves it via electrode C and infinity. It is not difficult to show that each of these electrodes will have the same field associated with it that it would have if in an infinite medium, except that it will be twice as strong. Along the center line the intensities of the three currents are thus given by the equations

\[ |\mathbf{J}_A| = |\mathbf{J}_B| = 2 \frac{1}{4\pi} \cdot \frac{1/2}{r^2 + d^2} = \frac{1}{4\pi} \cdot \frac{1}{r^2 + d^2} \tag{1} \]

\[ |\mathbf{J}_C| = 2 \frac{1}{4\pi} \cdot \frac{I_C}{d^2} = \frac{I_C}{2\pi} \tag{2} \]

\[ |\mathbf{J}_A + \mathbf{J}_B| = 2 \frac{1}{4\pi} \cdot \frac{1}{r^2 + d^2} \cos \theta = \frac{1}{2\pi} \cdot \frac{1}{r^2 + d^2} \cdot \sqrt{\frac{d}{r^2 + d^2}} \tag{3} \]

If \( \mathbf{J}_C \) is equal and opposite to this resultant, then

\[ \frac{I_C}{d^2} = \frac{1}{r^2 + d^2} \cdot \frac{d}{\sqrt{r^2 + d^2}} \tag{4} \]

That is

\[ \frac{1}{I_C} = (r^2 + d^2) \frac{d}{r^2 + d^2} \tag{5} \]

or

\[ \frac{1}{I_C} = \sqrt{\left(1 + \frac{r^2}{d^2}\right)} \cdot \frac{1}{r^2 + d^2} \tag{6} \]

This equation gives the fraction of the lead field current which must leave the conductor via the central electrode in order for there to be a null at a...
Fig. 8. Null point lead. (See appendix III)

depth $d$. The other portion $(1 - I_c)$ of this current leaves the conductor via the electrode at infinity.

If the central electrode and the electrode at infinity are connected to the positive terminal of the lead via resistors $R_c$ and $R_t$, respectively, then the unit current will divide according to the equation

$$I_c = \frac{1}{R_c} \frac{1}{R_c + 1/R_t} = \frac{R_t}{R_c + R_t}$$

providing that the resistors are very large in comparison with resistance between the electrodes. Substituting this in equation 6, the result is

$$\frac{R_c}{R_t} = \sqrt{\left(1 + \frac{\gamma^2}{d^2}\right)} - 1.$$ 

APPENDIX III

**Determining the General Direction of the Field from Catheter Measurements**

The general direction of the field within the heart can be estimated from three measurements if it is assumed that the field is uniform. In this case the voltage produced between two electrodes of the catheter will be given by the equation

$$V = pJ \cdot \mathbf{L} = pJ_y L_v + pJ_y L_T + pJ_y L_s$$

where $J_y$, $J_T$, and $J_s$ are the vertical, transverse and sagittal components of the lead field current density $J$; $L_v$, $L_T$ and $L_s$ are the corresponding components of the line $L$ separating the two electrodes of the catheter, and $p$ is the specific resistivity of the medium. This equation simply states that the voltage between the catheter electrodes will be proportional to (a) the projection of the lead field voltage gradient on the line joining the electrodes, and (b) the distance between the electrodes. If the orientation of the catheter is now changed, the equation will become

$$V' = pJ' \cdot \mathbf{L}' = pJ_y L_v' + pJ_y L_T' + pJ_y L_s'$$

where the prime symbols represent the new voltage and x-ray measurements resulting from the change in the orientation of the catheter. When the position of the catheter is changed for the third time, then

$$V'' = pJ'' \cdot \mathbf{L}'' = pJ_y L_v'' + pJ_y L_T'' + pJ_y L_s''.$$ 

Now, if the three voltages $V$, $V'$ and $V''$ are measured, and if $L_v$, $L_T$ and $L_s$, representing the positions of the catheter electrodes are found from side and frontal x-rays, these factors, combined with the measurable value of the specific resistivity $p$ of blood will enable the three simultaneous equations 1, 2, 3 to be solved for the three unknown components of the lead field $J_y$, $J_T$ and $J_s$. The equations that result are

$$I_c = \frac{1}{PD} \left[V(L_v L_s - L_v' L_s') - V(L_v' L_s - L_v L_s')\right]$$

$$I_T = \frac{1}{PD} \left[-V(L_v L_s' - L_v' L_s) + V(L_v' L_s' - L_v L_s)\right]$$

$$I_s = \frac{1}{PD} \left[V(L_v' L_s' - L_v L_s) - V(L_v L_s' - L_v' L_s)\right]$$

where

$$D = L_v(L_v L_s - L_v' L_s') - L_T(L_T L_s' - L_T' L_s) + L_s(L_s L_T - L_s' L_T')$$

**SUMARIO ESPAÑOL**

Este informe, el segundo de tres que tratan sobre el estudio de derivaciones electrocardiográficas principalmente desde el punto de vista eléctrico, contiene una breve discusión del número de técnicas experimentales y teóricas que pueden usarse para determinar el campo de una derivación dada. La importancia de estos campos fue señalada en el primer informe de esta serie.

Métodos para obtener patrones representando aproximaciones en dos dimensiones de los campos se consideran primero. Se demuestra como sencillos bocetos, basados en el modo de ver intuitivo del campo, pueden revelar prontamente las características generales de la derivación. Estos bocetos se usan para señalar los defectos de ciertas derivaciones que han sido usadas para obtener el componente sagital del vector cardiaco. Diseños especiales para estudiar campos en dos dimensiones, tales como "fluid mappers", se discuten brevemente.

Métodos para obtener información sobre los
campos en tres dimensiones se consideran después. El uso de modelos matemáticos sencillos se ilustra por medio del análisis de un nuevo tipo de derivación el cual es insensible a fuerzas electromotrices en ciertas posiciones. El procedimiento para calcular la dirección general del campo de las medidas de voltaje producidas entre los electrodos de un catéter se describe en detalle.

REFERENCES
10 Bryant, J. M.: Personal conversation.
Electrocardiographic Leads: II. Analysis
RICHARD MCFEE and FRANKLIN D. JOHNSTON

Circulation. 1954;9:255-266
doi: 10.1161/01.CIR.9.2.255

Circulation is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231
Copyright © 1954 American Heart Association, Inc. All rights reserved.
Print ISSN: 0009-7322. Online ISSN: 1524-4539

The online version of this article, along with updated information and services, is located on
the World Wide Web at:
http://circ.ahajournals.org/content/9/2/255