Electrocardiographic Leads

I. Introduction

By Richard McFee, M.S., and Franklin D. Johnston, M.D.

This paper is the first of a series of three which will deal with the relationship between the voltages in electrocardiographic leads and the electromotive forces of the heart. The general purpose of this series is to discuss various experimental and theoretic technics which can be used in the analysis of a given lead, and in the building up, or “synthesis,” of leads having desired characteristics. The procedures used are based on a fundamental theorem which takes into account not only the irregular shape and conductivity of the body but also the spatial dispersion of the electromotive forces within the heart. It is closely related to the “lead vector” concept of Burger and van Milaan.

In this first paper the basic definitions and theorems are developed. The second paper discusses various methods of analyzing leads. The third and last presents a number of systematic procedures for designing leads, both vectorcardiographic and unipolar. Such leads can have substantially higher accuracies than those now in use, because their design takes into account the shape and conductivity of the body and its tissues, and the eccentric and extended location of the heart.

FOREWORD

By Frank N. Wilson, M.D.

The basic principles which have led to a progressively better understanding of the factors that determine the form of the electrocardiogram were known over 30 years ago. Seventy years had passed since Helmholtz, with the experiments of DuBois Raymond in mind, stated and proved a number of theorems bearing on electrical currents in volume conductors. Waller’s work with the capillary electrometer in 1889, and his crude concept of the cardiac dipole, had been supplanted by the string galvanometer and the manifest vector of Einthoven and his associates. His, Tawara, Keith and Flack had furnished an accurate description of the specialized cardiac tissues, and Lewis had made an extensive and accurate study of the spread of the excitatory process over auricular and ventricular muscle, had introduced unipolar direct leads, and had advanced the theory of limited potential differences which amounted in essence to what was subsequently referred to as the “dipole hypothesis.”

Unfortunately, the application of the long known and well understood principles of potential theory to electrocardiography was not in general well received. Many of the more or less theoretic and mathematical papers along these lines aroused a storm of opposition. Some of the criticism came from physicians who felt that electrocardiography was a purely empiric science and that progress in the field could come only from comparison of the electrocardiographic findings with clinical and postmortem data. Much opposition came from the physiologists, many eminent in their field, who not only discounted any article of a theoretic nature but also regarded the dipole hypothesis as rank heresy.

Despite this opposition, the application of electrical principles to electrocardiography has continued, and the simpler approximations of the early years are now being replaced by more accurate and sophisticated analyses. It is astounding that almost all the recent theoretic advances are based on principles which were stated and proved by Helmholtz a century ago. His work was the result of his interest in the studies of Emil DuBois Raymond on injury currents of tissues. These were published in 1847. At the time Helmholtz wrote, too little was known about bioelectric currents to make his theorems of much practical importance in this field, and apparently they were forgotten before knowledge of this subject had advanced to the point where they were most needed.

The fundamental principles stated and explained by Helmholtz may be considered under three headings: (1) The Principle of Superposition, (2) The Principle of the Electromotive Surface and (3) The Principle of Reciprocity. The first of these has been the basis of the concept of the “lead vector” of Burger and van Milaan. The second has been used as a means of finding the field of an eccentric dipole in a sphere. Many of the applications of the third principle of Helmholtz, the “reciprocity theo-
rem,” are considered in the following three papers by McFee and Johnston. Their discussion of the implications of this basic theorem may prove to be one of the most interesting contributions to the understanding of the electrocardiogram that has been made in many years.

(This Foreword was written by Dr. Wilson in the winter of 1951 shortly after he had read and edited the first paper of the series. Before his death he had also read and commented on the second paper and had become familiar with a number of the topics taken up in the third. Many of the ideas presented in the papers were developed during discussions with him at his farm. There is no question that the work would not have been done had it not been for his invaluable suggestions, fellow interest and consistent and good humored encouragement.)

Our understanding of the manner in which the electromotive forces of the heart produce voltages in electrocardiographic leads has been greatly advanced in recent years through the introduction of the concept of the “lead vector” by Burger and van Milaan. This concept, which stems from basic electrical principles, has enabled us to consider, for the first time, the relation between a cardiac electromotive force and the voltage produced by it in a given lead, with due regard for the irregular shape and non-homogeneous electrical conductivity of the body. Electrocardiographic studies employing this concept are free from nearly all the objections which have been raised against mathematical studies which assume that the body is an infinite homogeneous conductor.

However, as might have been expected, the relation between the electromotive forces of the heart and the lead voltages has not been completely clarified by the concept of the lead vector. The main reason for this is that it assumes that the electromotive forces of the heart may be regarded as originating at a point and, of course, this is not actually the case. In fact, it is their dispersion that accounts for the special interpretation given to the chest leads. Burger and van Milaan were quite conscious of this, and drew special attention to it in their papers. Subsequent experiments by Wilson, Bryant and Johnston, and by den Boer have indicated that the influence of this dispersion is by no means negligible. However, the difficulty of dealing practically with this spread of the cardiac forces, of analyzing and building up leads with it in mind, has remained.

The method of studying electrocardiographic leads that is advanced in this series of papers is the outgrowth of an attempt to solve this problem. This has been done by replacing the concept of the lead vector with one closely related to it, that of the “lead field.”

The “lead field” is the electric field set up in the body when a unit current is introduced into the lead. The relation between it and the lead vector is an extraordinarily simple one. At any point in the heart the lead field is the lead vector with respect to electromotive forces arising at that point. The two concepts are similar in many respects. Just as Burger and van Milaan analyzed leads by finding the lead vectors, leads are analyzed here by finding the lead fields. Similarly, in place of the synthesis of leads having prescribed vectors, as is done in the universal vector-cardiograph of Becking, Burger and van Milaan, methods are worked out here for the synthesis of leads with prescribed fields.

It is a surprising fact that the concept of the lead field, although more general and useful than that of the lead vector, is nevertheless easier to understand and to use. The reason for this is that the lead field is not, like the lead vector, a mathematical concept. The studies on fluid mappers, recently reported by McFee, Stow and Johnston, demonstrate this quite clearly. The answers to many puzzling electrocardiographic questions become immediately obvious, without any mathematics, after one has grasped the basic significance of these fields in relating the voltages of the lead to the electromotive forces of the heart. It is only when one wishes to be exact that the necessity for the use of mathematics arises.

It is convenient to divide the presentation of this viewpoint into three parts: (a) definitions and theorems, (b) lead analysis, and (c) lead synthesis. A special attempt has been made to state the definitions and theorems in an unambiguous and clinically meaningful form, and to develop them in a compact and systematic way.

The first paper, in addition to deriving basic theorems concerning lead fields, shows how
the definitions of the "dipole moment" and the "potential," which arose in connection with the analysis of fields in infinite homogeneous media, may be applied to the human body. Following this, a simple definition of the accuracy of an interpretation of a lead is worked out. Using it, the difference between remote and local leads is discussed, in the hope of correcting the view, recently advanced by several authors, that the voltages produced in local leads are the same as those produced in remote leads, except that their magnitude is greater.

The second paper of this series concerns the analysis of leads in terms of their fields. Many experimental techniques for determining these "lead fields" will be pointed out, and practical examples given. The possibility of "null leads," which are insensitive to electromotive forces at some point within the heart, are touched upon. In addition, some of the outstanding electrocardiographic problems requiring the analysis of leads are discussed.

The third and last paper considers the problem of constructing leads having desired characteristics; that is, lead fields of certain types. Approximate, practical methods of designing such leads are worked out first, and used to find fairly simple vectorcardiographic leads, which are somewhat better than those now employed. These leads are constructed by connecting many electrodes to the two terminals of the lead by resistors of appropriate sizes. They are essentially a generalization of the resistor leads recently studied by Wilson and his associates. Simple improvements for the central terminal are also presented, along with methods for constructing variable leads (rotators) capable of selecting a desired component of the heart vector. In addition, the theoretic problem of obtaining "perfect" leads is treated in some detail. It is shown that they are possible in certain cases where the conductivity variations within the body are fairly simple. Finally, it is shown that the potential anywhere in or on the body (but outside the heart), as measured to a "perfect" indifferent electrode, can be found from measurements of the potential differences at the surface of the body, assuming only that the heart is spherical and homogeneous, and that the tissues outside the heart are homogeneous also.

It is quite interesting to note that the importance of the lead field, which suggested itself here as a means of generalizing the lead vector concept, was recognized and emphasized by Helmholtz over 100 years ago. The great significance of these fields is based upon the reciprocity theorem, whose potential importance in electrocardiography was emphasized recently by Wilson, Bryant and Johnston. In recent years the possibility of using such fields in studying electrocardiographic leads has suggested itself to several investigators in addition to the present authors. Lepeschkin, in his consideration of the "tubes or lines of flow of a lead" is actually dealing with lead fields. Isopotentials of lead fields have been traced not only by Wilson, Bryant and Johnston, but also by Schmitt. Schmitt was apparently the first to construct rotators, although similar and less elaborate circuits have been built independently not only by the author but also by McKay, Romans, Brody and Little.

The general purpose of this series of papers is to show how the basic principles of electrical theory may be used to study the relation between the electromotive forces of the heart and the voltages produced by them in electrocardiographic leads. The relations between these electromotive forces and the electrochemical and physiologic processes of the cardiac muscle, and between the latter and clinical diagnosis, although of equal importance, seem best treated as separate problems. For this reason, such considerations are omitted. It is to be hoped that a better understanding of all these relations will eventually result in substantial improvements in the reliability of electrocardiographic interpretations.

**Definition of a Lead and of an Electromotive Vector**

We are concerned here with the study of the relationship between the electromotive forces of the heart and the voltages produced by them in the different leads. Precisely what
do we mean by a "lead" and how can we describe an electromotive force?

The term "lead" was used by early electrocardiographic investigators to refer to a single pair of electrodes attached directly to the body. However, certain generalizations of this original use of the word have appeared in recent years. A central terminal chest lead, for example, has only one electrode attached directly to the body. The other terminal of this lead is connected to it only indirectly; that is, via resistors. In view of this use of the term, it would seem reasonable to define a lead as any pair of terminals, each connected to the body either directly or indirectly through any number of resistors.

However, even this broad use of the word might be objected to, because it does not include the "leads" which result from adding together the outputs of several vacuum tube amplifiers whose inputs are connected to electrodes attached to the body. What is the difference between leads of this type and the leads formed by connecting two terminals to the body via resistors? A simple theorem, which will be proved later, furnishes us with the answer. It states that no matter how many vacuum tube amplifiers are used, the same voltage can be obtained with a single "resistor lead" and one amplifier. This theorem insures that the following definition is a completely general one: A lead is a pair of terminals, each connected either directly or with any number of resistors to electrodes on the body. The resistances should be large enough that the connection of the lead to the body does not alter the electric field. To avoid ambiguity, one terminal of the lead will arbitrarily be called its "positive" terminal, and recording voltmeters should be connected so that their deflection has a positive sense when this terminal is electrically positive relative to the other.

The electromotive forces of the heart may be thought of either as electromotive surfaces such as exist between the metal and electrolyte in ordinary batteries, or as extended volume gradients such as occur when dissimilar solutions diffuse into one another. In any case, these two viewpoints are by no means incompatible; a volume gradient can be simulated by a large number of weak electromotive surfaces piled one on top of the other, and conversely, an electromotive surface can be treated as a very intense localized volume gradient.

So far as subsequent developments in these papers are concerned, it would not matter which of these viewpoints were chosen as a starting point. In fact, it would be possible to get along without either of them. This could be done by describing the electromotive forces "operationally," using miniature leads, in such a fashion that nothing is said or implied about the nature of the electromotive forces or the way in which they are generated. However, this approach, although rigorous, would be a very dreary one. For this reason, the most attractive approach has been chosen, and it is assumed here, purely for convenience, that the electromotive forces of the heart exist as electromotive surfaces.

One advantage to this assumption is that it fits in neatly with the theoretic work of Helmholtz on electromotive forces in volume conductors. Another advantage is that it is in agreement with the bulk of experimental and theoretic studies on cardiac electrophysiology. These studies indicate that the electromotive forces are located at the boundary between resting and active tissue, where they form an electromotive surface whose positive side is next to the resting tissue.

An electromotive surface of this sort may be described quantitatively by breaking it up into a number of small elements, each being essentially flat and having a uniform potential difference. The magnitude of the electromotive force of each of these elements can be specified in terms of the "electromotive vector" of the element, which is defined as follows:

The electromotive vector, \( \vec{E} \) of any small element of an electromotive surface is a vector which points in the direction faced by the positive side of the element and which has a magnitude equal to the potential difference across the element multiplied by its area.

Note that the dimensions of the components of this electromotive vector are volts times area. An element of an electromotive surface can be simulated in an electrolytic bath with
several pieces of nonpolarizing metal foil and a battery. The pieces of foil are placed back to back with a thin layer of insulation between them and each piece of foil is connected to a terminal of the battery. The net effect of an artificial element of this sort is indistinguishable from that of an element of a physiologic electromotive surface having the same area and potential difference.

**The Lead Field**

Using the specific definitions of a “lead” and “electromotive force” which have just been given, with the reciprocity theorem of Helmholtz, it is not difficult to see how the influence of the electromotive forces on the lead voltages may be found by studying the fields of the leads.

In his derivation of the reciprocity theorem, Helmholtz was considering the effect of an electromotive surface in a volume conductor on the deflection of a galvanometer connected to that conductor. He stated his theorem in the following way: “Every single element of an electromotive surface will produce a flow of the same quantity of electricity through the galvanometer as would flow through that element itself if its electromotive force were impressed on the galvanometer wire. If one adds the effects of all the electromotive surface elements, the effects of each of which are found in the manner described, he will have the value of the total current through the galvanometer.”

To see more clearly what this theorem means in electrocardiography, consider the effect of each individual element of an electromotive surface within the heart on the deflection of a galvanometer connected to a lead. This situation is illustrated by the example shown in figure 1A. Here $e$ is the potential difference across the element and $I$ is the current it produces in the lead. According to the reciprocity theorem, a battery with the same voltage put in series with the lead would cause the flow of the same current through the element. In this case, if the voltage of the battery were twice as great, naturally the resulting current also would be twice as great. In general, if the voltage of the battery in series with the lead is $E$ and the resulting current which flows through the element is $i$ (fig. 1B), then the reciprocity theorem requires that

$$E/e = i/I$$  \hspace{1cm} (1)

By transposing terms, this equation may be rewritten as

$$I = ie/E$$  \hspace{1cm} (2)

This last equation enables us to determine the current which flows through the electrocardiographic measuring apparatus as the result of the electric field of the element of the electromotive surface. However, since electrocardiographs are so calibrated that they measure voltage rather than current, it is desirable to rewrite equation 2 so that it furnishes the open circuit voltage, $v$, rather than the current, $i$, of the lead. To do this, one must first recognize that the current, $I$, (fig. 1A) is related to the open circuit voltage, $v$, by the equation

$$I = v/R$$  \hspace{1cm} (3)

where $R$ is the sum of the resistance of the galvanometer and the resistance seen “looking into” the lead. This same resistance, $R$, also relates the current, $I'$, (fig. 1B) which flows in

* The authors are indebted to Dr. Frank N. Wilson for this unusually clear and concise translation.
the lead to the voltage, \( E \), of the battery which produces it. The equation here is

\[ I' = \frac{E}{R} \quad (4) \]

If the values of \( I \) and \( E \) in these two equations are substituted into equation 2, then the \( R \) factor cancels, and

\[ v = e(i/I') \quad (5) \]

Finally, if the voltage, \( E \), is adjusted so that the current, \( I' \), flowing into the lead is unity, then this last equation becomes

\[ v = ei \quad (6) \]

and this is the equation we seek. It says that every element of an electromotive surface within the heart will produce an open circuit voltage, \( v \), in a given lead equal to the potential difference, \( e \), of this element multiplied by the current, \( i \), which passes through it as the result of connecting the lead to a unit source of current. If this current enters the negative terminal of the lead and leaves its positive terminal, then the voltage, \( v \), will be positive if the current through the element enters the negative face and leaves the positive.

When the effects of all the elements of the electromotive surfaces are added together, the following fundamental theorem results: The open circuit voltage, \( v \), produced in any lead is related to the electromotive forces of the heart by the equation

\[ v = e_1i_1 + e_2i_2 + \cdots \quad (7) \]

where the e's are the potential differences of the electromotive force elements, and the i's are the currents passing through the elements when a unit current is introduced into the lead. The sum is extended to include all electromotive force elements of the heart. As was just mentioned, the products \( ei \) here will be positive if the current leaves the positive face of the elements, and negative if it enters the positive face. This field of current, which is produced in the body when a unit current enters the negative terminal of a lead and leaves its positive terminal, will henceforth be called its “lead field.”

As an example of the practical value of this theorem, let us attempt to find the effect of a closed electromotive surface of uniform voltage on a lead whose electrodes are external to it. Since the potential differences of all electromotive force elements of this surface are the same, the voltage produced in the lead will, by equation 7, be equal to this voltage times the total lead field current which passes through the surface. But this current will be zero, since the lead electrodes are outside the closed surface, and any current entering the surface must, therefore, leave again somewhere else. Thus, we see that a closed electromotive surface of uniform potential difference will produce no voltage in a lead whose electrodes are all outside of it, and that this is true not only for the infinite homogeneous conductor, but also for any resistive conductor.

**The Lead Vector**

When a unit current is introduced into a lead, the resulting flow of current throughout the body will have a certain intensity and direction at every point in the body. That is, the flow of current forms a “vector field.” Its magnitude and direction at any point can be represented by the symbol \( \mathbf{J} \). (The magnitude of \( \mathbf{J} \) is here taken as the current density, in current per unit area, at the point in question.)

Now let us consider the relation between this vector field and the current passing through a specified element of the electromotive surface. If this element is sufficiently small, it will be effectively flat, and the magnitude and direction of the current passing through it will be the same at all points on it. The total current going through the element will then be equal to the component of the current field perpendicular to the surface of the element multiplied by the area of the element. Thus the voltage produced in the lead by that element will, by equation 6, be equal to the component of current perpendicular to the face of the element multiplied by the area of the element times its potential difference. If we now use the definition of the electromotive vector of such an element, which was previously given, and the mathematical definition of a “dot,” or scalar
product,* it follows that equation 6 can be rewritten as

\[ v = \mathbf{J} \cdot \mathbf{e} = J_x e_x + J_y e_y + J_z e_z \]  

where the subscripts indicate components in the \( x, y \) and \( z \) directions.

If this equation is now compared with equation 1 in the first paper on “Heart Vector and Leads” by Burger and van Milaan, it will be seen that the two, except for the symbols used, are identical. This means that the current field, \( \mathbf{J} \), at a point in the heart, resulting from the introduction of a unit current into the lead, has the same direction and intensity as the Burger lead vector of that lead with respect to electromotive forces located at that point.

Equation 7, it should be noticed, can now be rewritten as

\[ v = J_1 \cdot e + J_2 \cdot e + \cdots \]  

In this more general form, the lead equation 8 of Burger and van Milaan can be used to study all types of leads and distributions of electromotive forces, rather than just vector-cardiographic leads and electromotive forces at a single point. However, it is clear that much more insight into the nature of leads will be gained if we replace the algebraic concept of the lead vector with the essentially geometric and physical concept of the lead field. For example, in terms of the lead field, it is easy to see why electromotive forces having certain directions produce no voltage in the lead, and why they produce a maximum voltage when facing at right angles to these directions. Furthermore, the relation between lead vectors in different parts of the heart, formerly obscure, becomes quite obvious in terms of lead fields. For these and many other reasons, the idea of the lead field, rather than the lead vector, will be used in these papers as the basic tool for studying electrocardiographic leads.

**The Dipole Moment**

The concept of the dipole moment is often based on the analysis of the electric fields produced in an infinite homogeneous media at large distances from the charges or electromotive forces generating the fields. This analysis shows that although in the neighborhood of these forces the variations in potential may be quite irregular, as the distance from the forces increases these variations become much simpler, and at large distances the field becomes exactly identical to that which would be produced by a single electromotive element of appropriate magnitude and direction located within the surface enclosing the actual complex cluster of electromotive forces. This equivalent electromotive force is known as the “dipole moment” of the cluster.

Using the concept of the lead field, it is easy to show that such an equivalent electromotive force exists, and to find a formula for it. Consider a lead formed by one electrode remote from a cluster of electromotive surfaces in an infinite homogeneous conductor, and a second electrode at “infinity.” When a current leaves the conductor through the remote electrode and enters it through the electrode at infinity, the current lines in the neighborhood of the former will have the distribution shown in figure 2A. The small pear-shaped object in the figure is the region representing the heart, where the electromotive forces are located. It is clear that the flow lines of the current in this “heart” will have the appearance shown in figure 2B; that is, they will be for all practical purposes parallel. Since the resistance of the medium is uniform, this means that the current field throughout the “heart” will be uniform; that is, \( \mathbf{J} \) has an unvarying magnitude and direction. To see what this would mean, consider equation 9. If \( \mathbf{J} \) is a constant, this equation may be rewritten as

\[ v = \mathbf{J} \cdot (e_1 + e_1 + \cdots) = \mathbf{J} \cdot \mathbf{H} \]  

where

\[ \mathbf{H} = e_1 + e_2 + \cdots \]  

It is clear from equation 10 that a single electromotive force, of magnitude and direction \( \mathbf{H} \) located anywhere within the heart, would produce the same voltage in this lead as the actual complex cluster of electromotive forces. The same argument applies to a lead

* This product is discussed on pages 13, 14 and 16 of reference 2.
from any other remote point to infinity, the equivalent electromotive force in all cases being that given by formula 11. What this formula essentially states is that the dipole moment is equal to the sum of the electromotive vectors of all the elements of the electromotive surfaces. That is to say, the dipole moment tells us the direction and magnitude of all the electromotive vectors acting together. It is

defined as the product of the dipole moment and the side of the triangle representing the lead. Since the lengths of these sides are all the same, this product will be the projection of the dipole moment on the side multiplied by some constant equal for all the sides. This, of course, is the basic principle of the Einthoven triangle scheme.

If, on the other hand, the three electrodes are not arranged in an equilateral triangle, but nevertheless are remote from the heart, the fields produced by them will still be uniform within the heart, but will have varying magnitudes and directions. If the fields associated with leads between any two pairs of the electrodes are known, the field associated with the one remaining pair is also known, since it can be considered to be the superposition of the fields of the first two. For example, if the three lead electrodes are \( R, L \) and \( F \), and if the lead field associated with lead \( RL \) is \( \vec{J}_{RL} \) and the field associated with lead \( LF \) is \( \vec{J}_{LF} \), then the field associated with lead \( RL \) will be given by

\[
\vec{J}_{RF} = \vec{J}_{RL} + \vec{J}_{LF}
\]  

since the superposition of the lead fields of \( RL \) and \( LF \) will result in no current flowing into (or out of) the \( L \) electrode, a current of 1 ampere flowing out of the \( R \) electrode, and a current of 1 ampere flowing into the \( F \) electrode; that is, the introduction of 1 ampere into lead \( RF \). This means that the three lead vectors will add together in the fashion illustrated by the example in figure 2E, and that these three lead vectors will therefore form a triangle. This illustrates how the lead field concept leads to the “Burger” triangle.*

**The Heart Vector**

The electromotive forces of the heart, however, exist in a medium that is neither infinite

* This same argument applies regardless of the nature of the conductor or the location of the electrodes, provided that the lead vectors at only one point are considered.
nor homogeneous. Its resistivity varies not only from organ to organ, but also with the direction of the current flow. In these circumstances, then, how is it possible to use the concept of the dipole moment, and to find its components by measuring voltages at the surface of the body?

A great deal of confusion has arisen in connection with this problem. Attempts have been made to define a “heart vector” for the body in the same way that the dipole moment is defined for the infinite homogeneous medium; that is, in terms of the potential differences on the body surface remote from the heart. Unfortunately, it is a simple fact that there is no equivalent electromotive force which, located anywhere in the heart, will produce the same voltages at points remote from the heart that the actual electromotive forces do. For this reason, it is necessary to define the heart vector in some other way.

From the clinical point of view, the vector we are interested in is the same vector that would represent the dipole moment of the heart if its electromotive forces existed in an infinite homogeneous conductor. Of course they do not, but that does not prevent us from making the following definition. The heart vector is the dipole moment which would be associated with the electromotive forces of the heart if they existed in an infinite homogeneous conductor. That is, it is the vectorial sum of the electromotive vectors of the heart.

This definition may look at first like a rather useless one, since it is clearly impossible to remove the electromotive forces from the heart and place them in an infinite homogeneous conductor. However, our only purpose in doing this would be to measure the voltages produced there, and to use these measurements to calculate the dipole moment. This impossible operation might be bypassed then, if leads could be found whose fields in the heart are the same as those of leads in an infinite homogeneous conductor. That is, if the electromotive forces are the same and the lead field where they are located is the same, then by equation 9 the lead voltage will be the same. Since in an infinite homogeneous conductor a lead whose field within the heart is uniform will have a voltage produced in it proportional to the component of the dipole moment in the direction of the field, it follows then that any lead connected to the body whose field within the heart is uniform will have a voltage produced in it proportional to the component of the heart vector in the direction of the field, regardless of irregularities in the shape and conductivity of the body and of dispersion of the electromotive forces of the heart.

This fundamental principle is actually an immediate consequence of equation 9, but the above argument helps bring out its basic physical significance. In view of the irregular shape and conductivity of the body, it is of course by no means obvious that it is possible to find such uniform field or “heart vector” leads, and this is a question which has been investigated very carefully. These studies have shown that perfect leads of this sort can be constructed even when the outline and the variations in resistance of the body are taken into account, provided some simplifying assumptions are made concerning the latter. They will be described in the third paper of the series.

The Potential

The concept of the “indifferent electrode” also stems from the analysis of electric fields in the infinite homogeneous conductor. To understand how it can be applied to the body, despite the irregular shape and conductivity of the latter, it is first necessary to study the relation between the “potential” and the lead fields in an infinite homogeneous conductor.

The potential in such a conductor is defined as the voltage measured between the exploring electrode and a second electrode at infinity. At points remote from the heart this potential is proportional to the component of the dipole moment in the direction of the point, but this is not the case when the exploring electrode is near the heart. The reason for this is that the field of the lead formed by the exploring electrode and the electrode at infinity is then no longer uniform, but rather has the flow pattern of the type shown in figure 2F. The crowding together of the flow lines on the side of the “heart” toward the exploring electrode means that the lead field current is more intense there, and this in turn means that the voltage in the
lead is more sensitive to electromotive forces located there than to distant ones. Using the concept of the lead field, it is not difficult to derive an exact formula for the relation between the potential and these electromotive forces. It is

\[ v = \vec{J} \cdot \vec{e} = \left( \frac{1}{4\pi r^2} \right) \cdot e \]  

(13)

where \( \vec{1} \) is a unit vector directed along the line going from the electromotive force to the exploring electrode. This formula states that the current density of the current flowing into the exploring electrode decreases inversely with the square of the distance from it, and is directed towards it. The factor \( 4\pi \) is necessary because the area of a sphere of radius \( r \) about the electrode is \( 4\pi r^2 \).

![Diagram](image)

**Fig. 3.** See text

If the heart contains an electromotive surface which is almost closed, it is quite easy to locate the gaps in it by studying the potential about it. To see how this is done, consider the two distributions of electromotive force shown in figure 3A. The external voltages produced by these two distributions will be identical, because if the negative of one distribution is taken and added to the other, the result will be a closed surface having a uniform potential difference, and this will produce no voltage at all in any external lead, in accordance with a theorem previously proved. Thus, the small gaps in an almost closed electromotive surface will act like strong electromotive forces, and the surface itself will appear inert. This makes it easy to identify the gaps, and thus to identify infarcts, if they are the cause of the gaps.

It is not difficult to show with lead fields how the assumptions made by Einthoven justify the substitution of the central terminal for the electrode at infinity. He assumed that the heart and body are part of an infinite homogeneous conductor and that the three lead electrodes are situated in an equilateral triangle centered about the heart, which is supposed to be of negligible size in comparison with the dimensions of the triangle. In this idealized model, the current produced in the heart when the electrode at infinity is used can be thought of as the sum of two fields; the field of the exploring electrode as a sink, and the field of the electrode at infinity as a source. Since the latter is so far away, it produces no field in the heart, and the entire field is due to the current streaming into the exploring electrode. To show that the central terminal can be substituted for the electrode at infinity, then, it is necessary to show that equal amounts of current going into the three limb electrodes also produce no field at the center of the triangle. This is easily done. Since these electrodes are equidistant from the heart, and arranged in an equilateral triangle, the current fields they produce there will have the intensities and directions shown in figure 3B, and it is clear from the figure that the resultant of these three fields will be zero.

**The Indifferent Electrode**

The idea of the potential may be applied to the body in the same way as was the concept of the dipole moment. The clinician would like to have the voltage measured between the “indifferent electrode” and the exploring electrode identical to the potential which would exist at the exploring electrode if the electromotive forces of the heart were in an infinite homogeneous conductor. That is to say, he would make the following definition: an indifferent electrode is a reference electrode giving the same relative potential to the exploring electrode that it would have relative to infinity if it and the electromotive forces of the heart were part of an infinite homogeneous conductor. Since the voltage will be the same only if the lead fields are the same, it follows that an indifferent electrode is one which, together with the exploring electrode, produces a field within the heart which appears to radiate out from the exploring electrode in straight lines, and whose intensity there varies inversely.
with $4\pi$ times the square of the distance to the exploring electrode.

It is not difficult to construct an electrode of this sort. Various experiments have indicated that the central terminal is a good first approximation to one, provided that the exploring electrode is on the anterior chest. Methods of getting even better approximation are discussed in the third paper of this series. The possibility of perfect indifferent electrodes is also considered there.

It is important to realize that an indifferent electrode, as it is defined here, may be suitable for the exploring electrode at or near only one specific position. For example, although the central terminal seems to be a fairly satisfactory reference electrode when the exploring electrode is on the anterior chest, it is not when that electrode is on the back. It would be desirable to construct an electrode which would be indifferent regardless of the position of the exploring electrode, but this turns out to be completely impractical to do, even though it is theoretically possible.

**Measuring the Accuracy of a Lead**

The fields of practical leads never correspond exactly to the fields implicit in the idealized interpretations of them. The fields of heart vector leads, for example, are never precisely uniform, and the flow lines of indifferent electrode leads never precisely straight. In order to decide whether or not a lead is sufficiently accurate, it is desirable to have some quantitative index of its accuracy.

One way to do this is to compare the actual voltage produced in the lead with the voltage that would be produced in it if it were ideal. The actual voltage, $v$, will be in error by a certain percentage given in this particular case by

$$\epsilon = 100 \left| \frac{v - v_i}{v_i} \right|$$

(14)

where $v_i$ is the voltage that would be measured if the lead were ideal. It is natural to use a sort of average of this percentage as an index of the over-all error of the interpretation. The accuracy, $\alpha$, of the lead can be conveniently defined as 100 less the percentage error; that is,

$$\alpha = 100 - \epsilon$$

(15)

This accuracy will, of course, be dependent upon the location and the direction of the electromotive forces within the heart. For certain locations and directions of these, the accuracy may be zero; for others, 100 per cent. One can avoid having the concepts of error and accuracy of a lead dependent on the locations of the electromotive forces by supposing that the error and accuracy are defined for a random distribution of electromotive forces throughout the heart. This is a statistical concept which assumes that the electromotive forces are equally likely to occur at all points in the heart, and to have any direction.

To be specific, the error, $\epsilon$, of a given interpretation of a lead, in per cent, is defined as 100 times the ratio of the average magnitude* of the error voltage, $v - v_i$, to the average magnitude of the desired voltage, $v$, when these voltages are produced by a set of random distributions of electromotive forces within the heart. The accuracy of the interpretation, $\alpha$, is defined as 100 minus the percentage error.

It is easy to express this definition in terms of lead fields. This is a consequence of the fact that a given lead field can be thought of as the sum of two fields; the ideal field and the error field, which must be added to it to yield the actual field. The ideal field and the electromotive forces will determine $v$ and the error field and the electromotive forces, $v - v_i$. Since the average magnitude of the voltage produced by a random distribution of EMF's in any lead will be proportional to the average intensity of the lead field in the heart, it follows that equation 14 can be rewritten as

$$\epsilon = \left[ \left| \frac{J_i}{|A_V|} \right| \frac{J}{|A_V|} \right] 100$$

(16)

where $|J_i|_{AV}$ is the average intensity of the ideal field, and $|J|_{AV}$ is the average intensity of the error field which must be added to it to equal the actual field. That is, the error $\epsilon$ of a given interpretation of a lead is 100 times the ratio of the average magnitude of the error field to the average magnitude of the ideal field, where

*Since the electromotive forces are random, the same ratio would result if the root mean square values of the voltages were used, in place of the "average magnitude."
the error field is the field which must be added to the ideal field to equal the actual field.

As an example, let us consider the view, recently advanced by several authors, that the voltages on the chest are, for all practical purposes, proportional to the projection of the heart vector on the lead. How accurate is this interpretation of the chest leads?

To study this problem quantitatively we will need a model of the body. For simplicity, let us choose for this the infinite homogeneous conductor, assuming in addition that the heart is spherical in shape.

The mathematical procedure involved here (see Appendix for details) may be summarized as follows. The interpretation under consideration implies an ideal field in the heart which is uniform and parallel to the line joining the chest electrode to the center of the heart. The intensity of this ideal field is taken as equal to the vectorial average of the actual field, which for simplicity is calculated by assuming that the body is an infinite homogeneous conductor. The heart is assumed to be a sphere, and the second electrode of the lead is considered to be located at infinity. The average intensity of the error field, which is the difference between the actual and ideal fields, is then compared with the intensity of the ideal field, the exploring electrode being at a specified distance from the heart. The figure for the error of the interpretation, which results from taking the ratio of the average intensities of the error field and the ideal field, is then used to calculate the accuracy of the interpretation, using equations 15 and 16. This mathematical procedure leads to the graph of accuracy as a function of distance from the heart, shown in figure 4.

It is clear from this graph that the interpretation of chest leads in terms of the spatial vectorcardiogram is not likely to be very precise. The error ε here varies almost as a/R where a is the radius of the heart and R is the distance from the exploring electrode to its center. That is to say, when the exploring electrode is 2 heart diameters from the center of the heart, the error is about 25 per cent. When the distance is 1 diameter (half a diameter from the surface of the heart) the error is about 50 per cent.

Since the model of the body and the heart upon which this graph is based is obviously not an exact one, further studies of the same problem have been made using fluid mappers. In particular, the possibility that the high relative resistance of subcutaneous fat or the low relative resistance of the heart and blood would make the field substantially more uniform was investigated. These factors did tend to make the field slightly more uniform, but the change was by no means marked.

**Fig. 4.** Figure illustrating the accuracy of the assumption that the voltages in unipolar leads are proportional to the component of the heart vector in the direction of the exploring electrode. See text.

**Summary**

This paper is the first of three which will deal with the study of electrocardiographic leads from the electrical point of view. The general purpose of this series is to discuss various experimental and theoretical techniques which can be used in the analysis of a given lead and in the building up or “synthesis” of a lead having desired characteristics. The basic definitions and theorems which will be employed are developed in this first paper.

The most fundamental theorem relates the voltage produced in the lead to the electromotive forces of the heart by means of the “field” of the lead. The latter refers to the electric field set up in the body when a unit current is introduced into the lead. This theorem is closely related to the “lead vector” concept of Burger and van Milaan. It shows how the “dipole moment” and “potential” which would be associated with the electromotive forces of
the heart, if they were in an infinite homogeneous conductor, might be determined, despite the irregular shape of the body and the variation of the conductivity of its tissues.

In order to emphasize the relation of the idea of the lead field to previous studies of electrocardiographic leads, it is used in conjunction with Einthoven's assumptions to derive his triangle scheme, and to justify Wilson's substitution of the central terminal for the electrode at infinity. The Burger triangle is also discussed.

Because there will always be some difference between the interpretation of a clinical lead and its actual character, a quantitative method is developed for measuring the accuracy of such interpretations in terms of the fields of the leads. As an example, this method is used to test the view, recently advanced by several authors, that the voltages in the chest leads are due primarily to the component of the heart vector in the direction of the exploring electrode. It is shown that this view can be quite inaccurate when the electrode is close to the heart.

APPENDIX

The Accuracy of Considering the Voltages in Chest Leads Proportional to Components of the Heart Vector

The situation we are dealing with here, which involves approximations described in the text, is shown in figure 5. The solid lines in this figure are flow lines of the actual lead field, \( \vec{J}_a \), and the dashed lines are lines of the ideal field, \( \vec{J}_0 \), which corresponds to the interpretation being investigated. The equation for the actual field in polar coordinates is

\[
\vec{J}_a = 1_r / 4\pi r^2
\]

where \( \vec{r} \) is a unit vector pointing towards the exploring electrode. The intensity of the ideal field is assumed equal to the vectorial average of the actual field within the heart. A theorem in potential theory* states that this average is equal to the intensity at the center of the sphere, thus

\[
| \langle \vec{J}_a \rangle_r | = 1 / 4\pi R^2
\]

The components of this ideal field are given in polar coordinates by

\[
\vec{J}_i = (1_r, \cos \theta / 4\pi R^2) + (-1_r \sin \theta / 4\pi R^3)
\]

where \( \vec{1}_r \) is a unit vector perpendicular to the radius vector, which points in the direction of increasing \( \theta \).

The error field, \( \vec{J}_e \), is given by

\[
\vec{J}_e = \vec{J}_i - \vec{J}_i
\]

Substituting equations 1 and 3 into this, we have

\[
|J_e| = \frac{1}{4\pi} \sqrt{ \frac{1}{r^2} - \frac{2}{r^2} \cos \theta + \frac{1}{r^4} }
\]

or, expanding

\[
|\vec{J}_e| = \frac{1}{4\pi} \sqrt{ \frac{1}{r^4} - \frac{2}{r^2} \cos \theta + \frac{1}{r^4} }
\]

The average value of \( |\vec{J}_e| \) over the heart is given by the integral

\[
|\vec{J}_e|_{AV} = \frac{1}{(4/3)\pi a^3} \int_{R-a}^{R-a} \int_{0}^{\theta_{max}} |J_e| (2\pi \sin \theta) (r \, d\theta) \, dr
\]

The factor \((4/3)\pi a^3\) represents the volume of the sphere.

\( \theta_{max} \) here is given by the cosine law of oblique triangles

\[
a^2 = r^2 + R^2 - 2rR \cos \theta_{max}
\]

or

\[
\cos \theta_{max} = \frac{1}{2rR} (r^2 + R^2 - a^2)
\]

It is convenient to change the variable of integration from $\theta$ to $u = \cos \theta$. This transforms equation 8 to

$$
|J_\ell|_{AV} \sim - \frac{3}{8\pi a^2} \int_{R-a}^{R+a} r' \left\{ \left( \frac{1}{2\pi R} \right) (r'^2 + R^2 - a^2) \right\} \cdot \sqrt{\frac{1}{r'^2} - \frac{2}{r'^2 R^2} u + \frac{1}{R^4}} \, du \, dr
$$

(11)

The first integration here is easily accomplished with tables of integration, and the result can be put in the following form:

$$
|J_\ell|_{AV} = \frac{1}{8\pi a^2} \int_{R-a}^{R+a} \frac{r}{R} \cdot \left\{ \left( \frac{R^2 - r^2 + a^2}{R^2} \right)^{\frac{s}{2}} - \left( \frac{R^2 - r^2}{R^2} \right)^{\frac{s}{2}} \right\} \, dr
$$

(12)

Substituting for $r$ the variable $s$ defined by

$$
s = 1 - \frac{r}{R}
$$

(13)

equation 12 becomes

$$
|J_\ell|_{AV} = \frac{1}{4\pi R^2} \cdot \frac{1}{2} \int_{-k}^{k} \frac{s^3}{k^3} \cdot \left\{ \left( 2 - s \right)^s + \left( \frac{k^2}{s^2} - 1 \right) (1 - s)^s - \left( 2 - s \right)^s \right\} \, ds
$$

(14)

where $k$ is $a/R$.

The authors have been unable to find a way to express this integral in elementary functions, and

* An approximate value can be found by using the variables

$$
x = R - r \cos \theta; \quad y = r \sin \theta
$$

In this case equation 8 becomes

$$
|J_\ell|_{AV} \sim \frac{1}{4\pi R^2} \cdot \frac{1}{(4/3)\pi a} \int_{\text{circ}} \sqrt{a^2 - z^2} \cdot \sqrt{\frac{4}{R^2} \left( \frac{x}{R} \right)^2 + \left( \frac{y}{R} \right)^2 (2\pi y)} \, dy \, dx
$$

And the exact solution of this is

$$
|J_\ell|_{AV} \sim \frac{1}{4\pi R^2} \cdot 1.07 \frac{a}{R}
$$

That is

$$
\epsilon = \frac{100}{|J_\ell|_{AV}/|J_\ell|_{AV}} \simeq 107 \cdot \frac{a}{R}
$$

This simple equation gives values which are very close to those found by numerical integration.

for this reason have integrated it numerically, using the trapezoidal rule

$$
\int_a^b f(s) \, ds \simeq \frac{h}{2} \left[ f(a) + f(a + h) + f(a + 2h) + \cdots + f(b) \right]
$$

(15)

where $h$ is the spacing of the intervals.

In evaluating equation 14 the value of the integrand was calculated at 20 points evenly spaced in the interval $-k$ to $k$ for each specific $R$ considered. ($R = a/4, 8a$ and $16a$.) The resulting four values for $|J_\ell|_{AV}$ are the basis of the four points of the graph shown in figure 4. A slide rule was used in these calculations, and as a result of its limited accuracy and approximations involved in numerical integration the graph in figure 4 may be in error by several per cent.

**SUMARIO Español**

Este trabajo es el primero de una serie de tres que tratará sobre la relación entre el voltaje de las derivaciones electrocardiográficas y las fuerzas electromotrices del corazón. El propósito general de esta serie es discutir varias técnicas experimentales y teóricas que puedan ser usadas en el análisis de una derivación dada y en la construcción o síntesis de derivaciones de características deseadas. Los procedimientos usados son basados en un teorema fundamental que toma en consideración no tan solo la forma irregular y la conductividad del cuerpo pero si también la dispersión espacial de las fuerzas electromotrices dentro del corazón. Esto esta estrechamente relacionado al concepto de Burger y van Milaan de "vector de derivación."

En este primer trabajo las definiciones básicas y los teoremas se desarrollan. El segundo trabajo discute varios métodos de analizar las derivaciones. El tercero y último presenta un número de procedimientos sistemáticos para diseñar derivaciones vectorcardiográficas y unipolares. Estas derivaciones pueden tener exactitud substancialmente mas alta que las en presente uso, porque su diseño toma en consideración la forma y la conductividad del cuerpo y sus tejidos, y la localización excéntrica y extendida del corazón.
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Electrocardiographic Leads: I. Introduction
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Circulation. 1953;8:554-568
doi: 10.1161/01.CIR.8.4.554

Circulation is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231
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Print ISSN: 0009-7322. Online ISSN: 1524-4539

The online version of this article, along with updated information and services, is located on the World Wide Web at:
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