Improved accuracy of echocardiographic endocardial borders by spatiotemporal filtered Fourier reconstruction: description of the method and optimization of filter cutoffs


ABSTRACT The usefulness of digitized echocardiographic borders in quantitative regional left ventricular function analysis has been limited by the wide reported range for normal wall motion with this technique. We postulated that random error in endocardial border positioning is a major cause of this limitation. To test this hypothesis, we traced the endocardial borders field by field from 17 complete echocardiographic cycles in six dogs. These cycles showed a great deal of random movement, with each endocardial point reversing its motion an average of 18.5 times per cardiac cycle. Spatiotemporal Fourier analysis of these sequences demonstrated that most of the valid information on endocardial motion was contained in the first four temporal harmonics and the first seven spatial harmonics and that beyond these points the Fourier transform has the spectral characteristics of noise. Reconstruction of these 17 cycles eliminating all Fourier components above the sixth temporal and eighth spatial harmonics reduced the mean number of endocardial reversals per cycle to 2.3 (p < .00001). To derive the optimal temporal and spatial cutoffs, we compared reconstructions of each of the 17 cycles with three M mode echocardiograms obtained simultaneously with the cross-sectional images. Fourier cutoffs were varied between two and 20 harmonics and demonstrated that the optimal temporal cutoff was 5.5 harmonics and optimal spatial cutoff 6.9. With optimal filtering, the correlation between ventricular diameter derived from the M modes and from the cross-sectional images was \( r = .965 \), compared with .877 for the M mode vs unfiltered cross-sectional data (p < .0001). We conclude that two-dimensional filtered Fourier reconstruction significantly improves the accuracy of traced echocardiographic borders. This technique should be useful in the postprocessing of endocardial borders extracted by automated edge detection schemes and should also be applicable to cardiac images derived from modalities other than echocardiography.


STUDIES OVER the past 10 years have demonstrated that the two-dimensional echocardiographic (2DE) technique is potentially the ideal noninvasive method for the evaluation of regional left ventricular wall motion.\(^1\)–\(^8\) While the human observer is quite good at the qualitative interpretation of left ventricular echo-
cardiograms, several investigators have worked to develop more quantitative and objective analysis schemes.\(^9\)–\(^16\) These quantitative approaches typically require digitizing the position of the endocardial border at end-diastole and end-systole to calculate fractional radial shortening at specific points around the endocardium. Such methods assume that there is a definable normal range for radial shortening, so that segmental endocardial motion may be declared abnormal if it falls outside of this range. Unfortunately, studies of endocardial motion in normal humans\(^17\)–\(^19\) and dogs\(^19\) have found fractional radial shortening ranging from less than 0% (i.e., actual dyskinesia) to 100%. With such a wide “normal” range, not even a frankly akinetic segment would be graded as abnormal. Such findings are at odds with observations of left ventricular motion...
by implanted sonomicrometers,20, 21 M mode echocardiography,22 and radionuclide ventriculography,23 as well as with the general impression from clinical echocardiography (i.e., visual inspection) that normal ventricular contraction (particularly as with gross tricuspid regurgitation) imposes random error in the tracing of endocardial borders. Whether due to temporal misregistration of echocardiographic images, difficulty visualizing the endocardium on single frames, or shakiness tracing the endocardial contour, this error superimposes random noise on the spatiotemporal location of left ventricular borders, and may account for the 12% to 20% intraobserver variability reported in derived measurements.18,19

A method for filtering out this noise while leaving the underlying “true” ventricular motion unaffected would obviously be desirable. Signal analysis theory suggests that this noise (error) should be contained in random high-frequency fluctuations about the true signal, which predominantly contains low frequencies. Eliminating the high-frequency components from the observed signal thus should remove most of the noise. This filtration is most naturally achieved by Fourier (or spectral) analysis, which extracts from any signal the amplitude and phase of its component frequencies.24–26 Fourier transformation is best applied to a periodic signal, i.e., one that repeats in time or some other variable. Cross-sectional ventricular borders actually repeat in two dimensions: (1) time, represented by the duration of the cardiac cycle, and (2) space, represented by the angle around the left ventricular centroid, which repeats every 360 degrees. The Fourier transform may be applied to either one or both of these dimensions. Previously, Fourier analysis has been applied to the time course of cardiac movement20, 27, 28 and to ventricular shape,29 but it has not been applied to both the temporal and spatial dimensions simultaneously.

This study was undertaken, therefore, to investigate three issues: (1) Can two-dimensional Fourier transformation be applied to traced endocardial borders to yield a frequency spectrum that may be separated into signal and noise components? (2) Does elimination of these high-frequency noise components yield a truer rendition of ventricular wall motion? (3) Can we define the optimal temporal and spatial frequency cutoffs to yield the best reconstruction of ventricular wall motion?

Materials and methods

Animal preparation. Six mongrel dogs were used to provide echocardiographic images for this study. Under satisfactory general anesthesia with sodium pentobarbital, a midline sternotomy was performed, the pericardium was incised, and the heart was placed in a pericardial cradle. A water bath standoff was used in imaging.

Image acquisition. Cross-sectional echocardiographic images were obtained with an ATL Mark 600 mechanical sector scanner and a 3 MHz transducer and were stored on ½ inch videotape for further analysis. The following three parasternal short-axis echocardiographic planes were used in each dog: (1) that just above the tips of the papillary muscles (base), (2) that at the midpapillary level, and (3) that just below the base of the papillary muscles (apex). Images from three planes were obtained in all but one dog in which a technically adequate basilar view was not obtained. Thus, 17 2DE images were used in the analysis of endocardial motion.

As temporal “gold standards” for endocardial movement, three M mode echocardiograms were obtained for each imaging plane at separate points around the ventricular circumference and recorded on tape and strip chart. The first M mode (A) was recorded with the transducer positioned on the anterior surface of the heart with the M line directed from the anterior to the posterior walls through the center of the ventricle, a second M mode (B) was obtained by revolving the transducer as far medially as permitted by the chest activity with the M mode cursor still passing through the center of the left ventricular cavity (confirmed by 2DE guidance), and a final M mode (C) was obtained with the transducer as far laterally as possible, again with the M mode cursor still passing through the center of the chamber. Since each of these views had three associated M modes, 51 2DE/M mode pairs were available for optimization of the Fourier cutoff frequencies. Photographic prints showing the position of M lines A, B, and C relative to the sector scan were aligned with the use of either the papillary muscles or mitral valve insertions as internal landmarks. The angle between these M lines was then determined by protractor. If A were taken to be at 0 degrees, B was typically at +30 degrees, and C was at −30 degrees.

Data analysis. Cardiac cycles with optimal endocardial visualization were selected for digitization with a MicroSonics EasyView II Image Analyzer interfaced with a Digital Equipment Corporation VAX 11/780 as previously described.100 Endocardial borders were hand-traced field-by-field for the entire cardiac cycle; for each video field in the cardiac cycle, an array was constructed giving the radial distance of the endocardium from a floating centroid along 72 rays (at 5 degree intervals). These data were stored as r(t,θ), where t is time from the r wave and θ is angular distance around the endocardium. Since echocardiographic images were obtained at 60 fields per second, a heart rate of 90 beats/min, for example, resulted in an array, r(t,θ), of 40 × 72 real points giving the radial distance from the centroid in centimeters for each of these spatiotemporal points (arranged in a 16.67 msec × 5 degree grid).

At the time of initial tracing of the endocardial contours, the centroid of endocardial area was calculated for each video field. From these data, r(t,θ) may be calculated relative to either a fixed or floating centroid. In the canine preparation used for this study, there was little translation of the ventricle, so both centroid definitions gave highly similar reconstructions. For the endocardial ray reversal analysis and the Fourier cutoff optimization, a floating centroid was used. One effect of the floating centroid, however, was to make the first spatial harmonic very small (oppositely directed rays are symmetric). Thus, for the power spectrum calculation, a fixed centroid was used (see below).

Figures 1 and 2 show a typical sequence of traced endocardial borders. Figure 1 displays the borders in polar format with successive systolic (L) and diastolic (R) fields drawn concentrically. Figure 2 shows the same data in Cartesian coordinates.
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FIGURE 1. Polar display of raw echocardiographic data. Hand-traced endocardial borders from a single cardiac cycle at the midpapillary level in the dog are shown. Successive video fields (16.7 msec intervals) are drawn concentrically for systole (L) and diastole (R). Tick marks on horizontal and vertical axes are at centimeter intervals. The papillary muscles are identifiable at 2:00 and 4:00 in the tracings, but a considerable amount of random variation in endocardial positioning is evident.

FIGURE 2. Cartesian display of raw echocardiographic data. The data are identical to those in figure 1, but here successive frames are displayed sequentially along the x axis, with angle around the centroid (θ) displayed along the y axis. The z coordinate of each point is the distance of the endocardium from the centroid, r(t,θ). As in figure 1, while the coarse features of cardiac movement and shape are discernible, much is lost in the noisiness of the display.

with time along the x axis, angle around the centroid on the y axis, and distance of the endocardium from the centroid plotted along the z axis. Random (and presumably nonphysiologic) jitter is evident in both of these displays.

M mode echocardiograms were digitized at 1 msec intervals by tracing the anterior and posterior left ventricular endocardial borders with a digitizing tablet (Summagraphics Bitpad II) interfaced to the VAX computer. Left ventricular diameter (the distance between these two borders) was stored as m(t) for later comparison with two-dimensional reconstructions.

The symbols used in the text are listed in Appendix 1.

Fourier transformation. We performed two-dimensional fast-Fourier transformation on the periodic variables t and θ in the real array r(t,θ), resulting in a complex array of the same size, R(T,Θ), where T and Θ are harmonics of cardiac motion. Thus, R(1,0) is the magnitude of motion occurring once per cardiac cycle, R(2,0) represents movements that occur twice per cycle, R(0,2) represents deformations in the endocardial border that occur twice around the circumference (e.g., the papillary muscles), R(3,2) is the component of motion causing two deformations per circumference and three oscillations per cardiac cycle, etc. Figure 3 displays the logarithm of the magnitude of the spatiotemporal Fourier transform of the tracings in figures 1 and 2. Inspection of figure 3 reveals that the major spectral components are located at the low frequencies (in the center of the plot), with rapid fall off at the higher frequencies (especially so since this a logarithmic display).

Filtered Fourier reconstruction. It is important to realize that R(T,Θ) contains all of the information present in the physical data, r(t,θ). An inverse Fourier transform of the complex frequency data, R, will yield the original real image data, r, exactly. However, to test the hypothesis that most of the noise is contained in the higher frequencies, we performed a filtered reconstruction by first multiplying R(T,Θ) by a filter function F(T,Θ). We have used the Hanning function for filtering, which is 1 at T = Θ = 0 and falls smoothly to 0 for T > Tc or Θ > Θc, where Tc and Θc are the cutoff frequencies for the Fourier filter in the temporal and spatial (angle around the centroid) domains, respectively. Figure 4 shows a Hanning filter for Tc = 7 and Θc = 9, showing that such a filter sets approximately 95% of the Fourier components to 0. We use F(T,Θ) to refer to a general filtering function, whereas F(T,Θ) (T,Θ) denotes the specific filter with use of Fourier cutoffs of Tc and Θc.

Multiplying each point in R(T,Θ) by the corresponding point in F(T,Θ) yielded a filtered complex array, S(T,Θ), with its low-frequency components unchanged from R and its components above Tc and Θc set to 0. S(T,Θ) then underwent inverse Fourier transformation yielding s(t,Θ), a smoother version of the original image r. By varying Tc and Θc it was possible to adjust the degree of smoothing in the temporal and spatial domains independently. Notationally, s(T,Θ) (t,Θ) is the specific filtered
reconstruction resulting from cutoffs $T_c$ and $\Theta_c$. Appendix 2 presents some mathematical principles of Fourier filtration.

**Left ventricular diameter extraction.** The filtered and unfiltered 2DE data were compared with the original M mode echocardiograms by use of left ventricular diameters calculated from the two modalities. The time course of left ventricular diameter was derived from both the filtered and unfiltered two-dimensional arrays by adding together oppositely directed endocardial rays, e.g., $r(t, \theta') + r(t, \theta' + \pi)$, where $\theta'$ is the incident angle of the corresponding M mode echocardiogram. The linear correlation coefficient was calculated for this diameter compared with $m(t)$, the diameter calculated from the M mode echocardiogram. For shorthand, we denote the $\theta'$ left ventricular diameter from $s^{T, \Theta}_{C, I}(t, \theta')$. Note also that $s^{T, \Theta}_{C, I} = r$ since the cutoff frequencies are set infinitely high so all frequencies are reconstructed.

**Data analysis protocol.** Data analysis was performed in the three following stages to address the questions of the study.

**Spatial and temporal power spectra.** For calculation of the spatial and temporal power spectra, the Fourier transforms of 14 of the 17 2DE images were used with $r(t, \theta)$ taken relative to a fixed centroid. The other three images (all apical views) had very small end-systolic areas that fell outside the fixed centroid and so could not be described in polar geometry. The amount of power at a given frequency is proportional to the squared magnitude of the Fourier transforms at that frequency. Since $R(T, \Theta)$ was a function of both temporal and spatial frequency, we summed the squared Fourier magnitude over $T$ to yield power as a function of $\Theta$ and over $\Theta$ to yield power as a function of $T$:

\[
P(T) = \sum_\Theta |R(T, \Theta)|^2
\]

\[
P(\Theta) = \sum_T |R(T, \Theta)|^2
\]

Before this summation, the mean radius, $\bar{r}$, was subtracted from $r(t, \theta)$, so the power spectra would reflect the motion and distortion in $r(t, \theta)$ and not mean values. To adjust for changes in animal size, $P(T)$ and $P(\Theta)$ were normalized so that the sum over all harmonics would be 100%. Their logarithms were plotted and inspected visually for evidence of a high-intensity peak at low frequencies representing the true data along with a much broader, lower amplitude curve, representing random noise, that was evident at higher frequencies. Average $P(T)$ and $P(\Theta)$ for the 14 images were compared harmonic by harmonic to each other by paired $t$ test to evaluate the relative shapes of the temporal and spatial power spectra (central peak) and the relative magnitude of noise (broad tail) in the temporal and spatial domains (i.e., errors occurring between fields vs within individual fields).

**Comparison of endocardial ray reversals.** As an initial assessment of the smoothing in left ventricular wall motion with Fourier filtration, we reconstructed the 17 short-axis views with $T_c = 7$ and $\Theta_c = 9$. For each reconstruction, the number of endocardial motion reversals was counted for each ray and averaged around the circumference. This number was compared by paired $t$ test with the corresponding mean number of endocardial motion reversals from the original traced contours, $r(t, \theta)$.

*Optimization of $T_c$ and $\Theta_c$ by comparison with M modes.* Original M modes, digitized at 1 msec intervals and smoothed with a three-point moving average, were resampled at 16.67 msec intervals to allow direct comparison with derived left ventricular diameters extracted from the raw or filtered two-dimensional data. Video prints of the M line superimposed on the two-dimensional echocardiogram were used to measure $\theta'$, the ray number of the two-dimensional echocardiogram corresponding to the given M mode. To allow for slight temporal misalignment of the 2DE and M mode data sets, the correlation coefficient between $m(t)$ and $r(t, \theta')$ was calculated as $r(t, \theta')$ was shifted a maximum of two video fields (33 msec) forward or backward in time. The data sets were subsequently held in this maximally correlated shift. This raw correlation coefficient was stored as $\text{Corr}(\omega_c, \omega)$ and used as the standard against which to judge the correlation of $m(t)$ with the filtered two-dimensional echocardiogram.

For $T_c = 2$ to 20 and $\Theta_c = 2$ to 20, multiple $s^{T_c, \Theta_c}(t, \theta')$ were generated by multiplying $R(T, \Theta)$ by $F^{T_c, \Theta_c}(T, \Theta)$, then performing the inverse Fourier transformation. The left ventricular diameter corresponding to the $\theta'$ line, $s^{T_c, \Theta_c}(t, \theta')$, was then correlated with $m(t)$ and stored in an array as $\text{Corr}(T_c, \Theta_c)$. Optimal filtration parameters were taken to be the $T_c$ and $\Theta_c$ that

![Figure 3](Figure3.png)

**FIGURE 3.** Fourier magnitude of raw echocardiographic data. Shown is the logarithm of the Fourier transform magnitude of the data in figures 1 and 2. Temporal frequency ($T$) is shown along the $x$ axis ranging between $-29$ and $+29$ cycles per RR interval; spatial frequency ($\Theta$) is along the $y$ axis, ranging from $-35$ to $+35$ cycles per endocardial circumference. Although there is a prominent central peak at the low frequencies near $T = \Theta = 0$, there remains significant magnitude even at the highest frequencies, representing most of the random noise seen in figures 1 and 2.

![Figure 4](Figure4.png)

**FIGURE 4.** Hanning Fourier filter. The Fourier transform in figure 3 is multiplied by this function before performing the inverse Fourier transform to yield a smoothed version of figures 1 and 2. In this example, the temporal cutoff, $T_c$, is 7, and the spatial cutoff, $\Theta_c$, is 9. All frequencies outside the range $T = -7$ to 7 and $\Theta = -9$ to 9 are set to zero. At $T = \Theta = 0$, the Fourier component is unaffected (i.e., is multiplied by 1), then falls smoothly to 0 as $T$ and $\Theta$ approach the cutoff frequencies.
Temporal and spatial power spectra. The average power per Fourier harmonic is shown for the temporal domain, representing the motion of an average point on the endocardium, and the spatial domain, representing the shape of the endocardium for an average video field. Data are mean ± SEM for 14 ventricular transforms. In each case the mean radius was subtracted from r(t,θ), and the power per harmonic displayed as a percentage of the total power for all harmonics. * and # signify harmonics at which the spatial and temporal spectra differ significantly. This demonstrates that the bulk of the meaningful motion information is concentrated in the lower (<4) harmonics, that shape information requires more harmonics (about 7), and that the level of temporal noise (error in tracing between curves) is greater than the error within a single curve.

yielded the maximal correlation coefficient, Corr(T_{opt},\Theta_{opt}). To test the significance of the improvement from Corr(\infty,\infty) to Corr(T_{opt},\Theta_{opt}), we performed Fisher's z transformation on these coefficients to yield z_0 and z_1, respectively. The significance level is then

\[ p = \text{erfc} \left( |z_1 - z_0| \sqrt{\frac{3}{2N}} \right) \]

where N is the number of video fields in the reconstruction and erfc is the complementary error function. A similar optimization protocol was also done for the time derivatives of ventricular diameter, dm/dt vs ds_{T,\Theta}^T(t,\Theta)/dt.

Computer programs. We used the IMSL Fortran subroutine FFT3D to perform the two-dimensional direct and inverse fast-Fourier transformations. This algorithm is particularly useful since it does not require that the input array dimensions be a power of two in length. The Unified Graphics Library on the VAX 11/780 was used to generate figures 1, 6, and 10. The GraphiC Subroutine Library called from the C language on a microcomputer (Scientific Endeavors Corporation, Kingston, TN) was used for figures 2, 3, 4, and 7.

Results

Spatial and temporal power spectra. Figure 5 shows the average power spectra in the temporal and spatial domains for 14 left ventricular two-dimensional Fourier transforms. Each curve is normalized so that the sum of all harmonics (in each domain) is 100%; the logarithm of this relative power is displayed. The spatial spectrum is derived from the average shape of the ventricle throughout the cardiac cycle, whereas the temporal spectrum is derived from the motion of an average endocardial point throughout the cardiac cycle. It is evident that the spatial and temporal Fourier spectra produce very different curves. In particular, the temporal spectrum has most of its power in the 0 and 1 harmonics and then drops very quickly. After the fourth harmonic, however, the temporal spectrum becomes quite flat, remaining at about the 1% level to the thirteenth harmonic.

FIGURE 5. Temporal and spatial power spectra. The average power per Fourier harmonic is shown for the temporal domain, representing the motion of an average point on the endocardium, and the spatial domain, representing the shape of the endocardium for an average video field. Data are mean ± SEM for 14 ventricular transforms. In each case the mean radius was subtracted from r(t,θ), and the power per harmonic displayed as a percentage of the total power for all harmonics. * and # signify harmonics at which the spatial and temporal spectra differ significantly. This demonstrates that the bulk of the meaningful motion information is concentrated in the lower (<4) harmonics, that shape information requires more harmonics (about 7), and that the level of temporal noise (error in tracing between curves) is greater than the error within a single curve.

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FIGURE 6. Filtered reconstruction of echocardiographic data. Shown here is the smoothed reconstruction of the data in figure 1 with cutoff frequencies T_{c} = 7 and \Theta_{c} = 9. To generate this image, the raw Fourier transform displayed in figure 3 was multiplied by the filter function in figure 4 and then underwent inverse Fourier transformation. As in figure 1, systolic borders are on the left and diastole shown at right.
By contrast, the spatial spectrum has a broader central peak, with significantly more power than the temporal spectrum in the second through fifth harmonics. Beyond the sixth harmonic, however, the spatial power falls below the temporal, and this relationship persists to the limit of the Fourier transform. This high-frequency spatial tail is flatter than the central peak but not nearly so flat as the temporal tail, continuing to drop out to the limits of the power spectrum.

Reduction in endocardial motion reversals with Fourier filtration. Figures 6 and 7 show the filtered Fourier reconstruction of the data displayed in figures 1 and 2, with Tc = 7 and Θc = 9 (i.e., those obtained with the Hanning filter from figure 4). As in figures 1 and 2, figure 6 displays the data in polar format with systole to the left and diastole to the right, whereas figure 7 uses Cartesian coordinates to display the same data. It is visually obvious that both the endocardial surface and its motion are considerably smoother in the filtered reconstruction than in the original raw data. To quantify this smoothing, the number of endocardial motion reversals throughout the cardiac cycle were counted for each ray. Figure 8 shows a typical time course of endocardial distance along two oppositely directed rays (displayed in the format of an M mode echocardiogram). The top panel is derived from the raw data in figure 1, whereas the bottom panel displays the ray motion after smoothing similar to that in figure 6. For 14 raw data sets, there was an average of 18.5 ± 1.0 (mean ± SEM) endocardial motion reversals along a given ray. After filtration with Tc = 7 and Θc = 9, the reconstructed data had 2.3 ± 0.1 ray reversals (p < .0001).

Comparison with M mode echocardiograms and optimization of Tc and Θc. Left ventricular diameters from 17 two-dimensional echocardiograms were each compared with three M mode echocardiograms obtained simultaneously, corresponding to the central, medial, and lateral M lines through the sector scan. These 51 2DE/M mode pairs were compared under various degrees of Fourier filtration. The values of Tc and Θc explored for each pair ranged from as low as two to as high as 20.

The correlation coefficient between the raw 2DE data, s^\(t\)'(t, θ'), and the left ventricular diameters derived from the true M mode images, m(t), averaged r = .877 ± .007 for the 51 pairs. Comparing the temporal derivations ds^\(t\)'(t, θ')/dt with dm(t)/dt yielded a significantly (p < .0001) lower r value (.310 ± .016) due to the tendency of differentiation to magnify noise.

Figure 9 is a typical matrix showing the correlation between m(t) and s^\(Tc, \Theta c\) (t, θ') for Tc = 3 to 11 and Θc = 6 to 12. The raw correlation coefficient, Corr(\(\infty, \infty\)), for this example is .827, and improved correlation is seen at all values of Tc and Θc. The best correlation, .984, is seen at Tc = 3 and Θc = 8. This correlation peak is very broad, however; the use of Tc between 2 and 7 and Θc between 6 and 10 yields r = .95 or greater.

Figure 10 shows graphically the change in the shape of s^\(Tc, \Theta c\) (t, θ') for various Tc with Θc = 8. The true M mode data are the top curve; the raw 2DE data,
CORRELATION OF 2DE VS M-MODE DIAMETERS

\[
\begin{array}{cccccccccc}
& 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & \infty \\
2 & 0.965 & 0.969 & 0.971 & 0.973 & 0.972 & 0.972 & 0.969 & 0.969 & \\
3 & 0.975 & 0.981 & 0.983 & 0.983 & 0.984 & 0.983 & 0.982 & 0.981 & \\
T_c4 & 0.975 & 0.979 & 0.980 & 0.980 & 0.979 & 0.979 & 0.975 & 0.974 & \\
5 & 0.971 & 0.972 & 0.972 & 0.971 & 0.970 & 0.967 & 0.964 & 0.963 & \\
6 & 0.966 & 0.966 & 0.965 & 0.964 & 0.964 & 0.960 & 0.956 & 0.955 & 0.953 \\
\end{array}
\]

0.827

CORRELATION OF 1ST TEMPORAL DERIVATIVES

\[
\begin{array}{cccccccccc}
& 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & \infty \\
2 & 0.828 & 0.826 & 0.834 & 0.838 & 0.837 & 0.838 & 0.822 & 0.833 & \\
3 & 0.857 & 0.871 & 0.871 & 0.869 & 0.877 & 0.873 & 0.871 & 0.872 & \\
T_c4 & 0.852 & 0.854 & 0.853 & 0.858 & 0.858 & 0.848 & 0.846 & 0.850 & \\
5 & 0.827 & 0.822 & 0.819 & 0.819 & 0.821 & 0.814 & 0.805 & 0.809 & \\
6 & 0.802 & 0.792 & 0.784 & 0.784 & 0.773 & 0.766 & 0.763 & 0.754 & \\
\end{array}
\]

0.229

FIGURE 9. Optimization of \( T_c \) and \( \Theta_c \). Top, The correlation matrix comparing left ventricular diameter derived from an actual M mode echocardiogram and diameter derived from a reconstructed two-dimensional echocardiogram filtered with temporal cutoff, \( T_c \), varying from 2 to 6, and spatial cutoff, \( \Theta_c \), varying from 4 to 11. Bottom, The correlation between the time derivative of the left ventricular diameter derived from the M mode and filtered two-dimensional data. The correlation with the unfiltered data (equivalent to filtering with \( T_c = \Theta_c = \infty \)) is shown at the lower right of each matrix (underlined). The optimal filtering characteristics for this data set were found to be \( T_c = 3 \) and \( \Theta_c = 8 \) (boldface), as assessed both by left ventricular diameter and its time derivative. The improvement in correlation for left ventricular diameter, 0.827 to 0.984, was significant at the \( p < .002 \) level; improvement in correlation for the time derivative was significant at \( p < .003 \).

\( s^{\infty,\infty}(t,\theta') \), are the second curve; filtered versions of this 2DE data are shown subsequently with the corresponding \( T_c \) and \( \Theta_c \) to the left and Corr(\( T_c,\Theta_c \)) to the right. Correlation of the temporal derivatives is shown at the far right.

For 51 2DE/M mode pairs, optimal two-dimensional Fourier filtration improved the correlation from \( r = 0.877 \pm 0.007 \) to \( 0.965 \pm 0.004 \) (\( p < .0001 \)). Correlation between the temporal derivative of these data showed even greater improvement, from \( r = 0.297 \pm 0.014 \) to \( 0.816 \pm 0.017 \) (\( p < .00001 \)). The optimal temporal cutoff, \( T_{opt} \), for these reconstructions was 5.5 ± 0.3 harmonics; the optimal spatial cutoff, \( \Theta_{opt} \), was 6.9 ± 0.2 harmonics, significantly (\( p < .01 \)) greater than \( T_{opt} \), although as noted above the individual peaks in the correlation matrixes tended to be quite broad. In particular, it was observed that Corr(\( T_c,\Theta_c \)) rose and fell fairly sharply with variations in \( T_c \) (typically within three or four harmonics), whereas variation with \( \Theta_c \) was much more gradual. This is not surprising since \( m(t) \) is a temporal gold standard that would be expected to have more discriminating power over \( T_c \) than \( \Theta_c \).

Discussion

We have described a new technique, two-dimensional filtered Fourier reconstruction, which we have used to reduce the random error in traced endocardial borders from cross-sectional echocardiograms. We have shown that this is a plausible approach since our calculated temporal and spatial power spectra both demonstrate low-frequency regions in which the true data predominate and high-frequency regions characteristic of random noise. Eliminating these high-frequency data by filtration before inverse Fourier transformation has yielded smoother ventricular shapes and motion that are more in keeping with observations made by other modalities. These reconstructed images have also been shown to be quantitatively more accurate by comparison with M mode echocardiograms, which has also allowed selection of the optimal frequency cutoffs in the Fourier filter.

Spatiotemporal Fourier transform of left ventricular motion. Fourier analysis is used in many scientific fields to extract the frequency content from an observed physical signal. It may be applied to functions of either time
or space and is particularly appropriate for periodic signals. Several authors have examined the Fourier transform of the time course of left ventricular motion, and one group has used the Fourier transform to analyze the shape of the left ventricle on the two-dimensional echocardiogram. It is a natural theoretical extension to extract both the temporal and spatial frequency content simultaneously from a series of endocardial contours spanning the cardiac cycle as we have done here. Such an analysis provides us with the temporal and spatial power spectra from the same data (figure 5), demonstrating very distinct differences between the spatial and the temporal curves. In particular, it shows that the bulk of the temporal spectrum is contained in the first four harmonics, as observed by Bachrach and Rankin and their colleagues. Since endocardial motion should contain just one inward and outward motion per cardiac cycle, one might expect that only the first harmonic would be significant. Including only one harmonic, however, would require that systole and diastole be of equal length. To have unequal systolic and diastolic periods requires higher harmonics. Although Heusch et al. observed that analysis of the first temporal harmonic alone could distinguish between normal and ischemic wall thickening as assessed by sonomicrometry, they also demonstrated that considerably better fidelity with the original signal was achieved by including the first five harmonics.

The spatial spectrum shows a significantly broader central peak, with power falling to 1% of the total only after the sixth harmonic. This reflects the more complex endocardial shape, including the two papillary muscles. As above, one might think that only two harmonics would be required to reconstruct the papillary muscles. However, higher harmonics are required since

![Diagram](http://circ.ahajournals.org/)

**FIGURE 10.** Display of left ventricular diameter from M mode and raw and filtered two-dimensional data. This sequence of curves shows the time course of left ventricular diameter derived from M mode echocardiograms (top), from raw two-dimensional data (second curve), and from filtered reconstructions with $\Theta_c = 8$ and $T_c$ ranging from 2 to 13. This is the same analysis displayed in matrix form in figure 9. The correlation coefficients are at the right, with the correlation for diameter in the first column and that for the temporal derivative in the second column. The optimal correlation is found at $T_c = 3$.

![Diagram](http://circ.ahajournals.org/)

**FIGURE 11.** Idealized power spectrum for noisy data. Most true data signals will have a Fourier power spectrum with a high central peak and rapid falloff, whereas random (Gaussian) noise has a much lower central peak but significantly slower falloff so that at higher frequencies, it is much higher in power than the true data. The sum of these two curves (shown here on a logarithmic scale) is concave upward where the noise and data spectra cross. It is here that the Fourier cutoff should be applied for filtered reconstruction.
LOW LEVEL (PIXELS)
- Raw echo image
- Spatiotemporal smoothing
- Edge enhancement
- Edge extraction
- Extracted features

HIGH LEVEL (FEATURES)
- Extracted features
- Fourier filtering of periodic features
- Expert system or operator analysis
- Anatomic interpretation

FIGURE 12. Stages in automated echocardiographic processing. Low-level processing is analogous to the function of the human retina, converting millions of incident pixels per second into several thousand features per second that are then analyzed by high-level processing analogous to the occipital cortex. The Fourier filtration of periodic features described in this article overcomes many of the limitations of current approaches to low-level processing, correcting for random error in the spatiotemporal localization of these features.

FIGURE 13. Demonstration of Fourier filtration. Idealized time course of ventricular diameter (A) has a Fourier transform (B) with most power in the lowest frequencies. Computer-generated random error (C) has a flat Fourier transform (D). Their sums (E and F) are typical of noisy data. Multiplying F by the Hanning filter in H and then performing the inverse transformation yields G, which almost replicates the underlying "true" data in A.
the power spectrum from noisy data that might be observed in the laboratory (the logarithmic scale distorts the appearance of the addition somewhat). At low frequencies, the observed power spectrum is dominated by the true data, whereas the higher frequency components arise almost completely from the noise. Dividing these two zones is the point at which the DATA and NOISE spectra cross, which is also where the "OBSERVED POWER SPECTRUM" is most strongly concave upward. The cutoff frequency of the Fourier filter would be set here to achieve optimal noise reduction. The critical point is that one need not know the exact shape of either the true data spectrum or the pure noise spectrum to define the cutoff frequency. Analysis of the observed data spectrum alone provides a reasonable guide to the optimal cutoff frequency.

Inspection of figure 5 shows that both the spatial and the temporal spectra may be separated into signal and noise regions. The temporal spectrum drops off quite sharply by the fourth harmonic, then levels out at around 1% of total power, dropping slowly thereafter. These clearly represent two different curves. That the noise region is so nearly flat is compelling evidence that the temporal noise is indeed distributed as a Gaussian random variable (i.e., errors made in tracing one contour are independent of errors tracing subsequent ones).

The spatial spectrum is somewhat less distinct but still separable into a rapidly falling central peak and a more slowly falling noise tail with a point of maximal upward curvature between the two segments occurring around the seventh harmonic. Beyond this point the level of spatial noise (intraframe) is significantly less than the temporal noise (interframe). Moreover, the fact that it continues to decline at higher frequencies implies that spatial noise is not entirely random between successive rays, i.e., that an error in placing one ray may cause a similar error in placing the adjacent rays.

Thus, on the basis of spectrum shape alone, one would predict that eliminating Fourier components above approximately the fourth temporal and seventh spatial harmonics before performing inverse transformation should eliminate most of the random noise while preserving most of the valid motion data. Since temporal noise is greater than spatial, one would expect most of the improvement to arise from temporal filtering.

Assessment of filtering by endocardial ray reversals. It is almost axiomatic that normal left ventricular contraction is a smooth, efficient process. Thus, one would expect two endocardial ray reversals per cardiac cycle, at end-diastole and end-systole. The raw two-dimensional echocardiographic data demonstrated over 18 such ray reversals per cardiac cycle, which is surely nonphysiologic. With even a moderate degree of Fourier filtration \((T_c = 7\) and \(\Theta_c = 9\)), this number was reduced quite close to the predicted ideal, supporting the improved accuracy obtained with the filtering process.

Determination of optimal \(T_c\) and \(\Theta_c\) by comparison with M mode results. While the reduction in endocardial ray reversals makes a strong argument for Fourier filtration, it is largely a qualitative one. The comparison with actual M mode data provides quantitative evidence that this noise reduction is actually resulting in a more accurate representation of cardiac motion, not just one that looks better. Additionally, it provides an empiric basis for choosing the best temporal cutoff and, to a lesser extent, spatial cutoff for the filtering function. The M mode data have a temporal resolution of 1 msec, allowing them to be a gold standard against which to compare the left ventricular diameters extracted from the two-dimensional data sampled at 16.7 msec intervals. By these criteria, \(T_{opt}\) was found to be between 5 and 6. This agrees quite closely with the observation that the break point between the noise and data curves in the temporal spectrum occurs between the third and fifth harmonics. (Note that if \(T_c = 6\), then the Hanning filter will set all harmonic components above 5 to zero, with the third and fourth harmonics reduced in magnitude by 50% and 75%, respectively.)

The level of \(T_c\) and \(\Theta_c\) always represents a compromise between including as much of the true data spectrum as possible and eliminating as much noise as possible. Regardless of where the cutoff might be placed in figure 11, there will always be some legitimate high-frequency data that will be thrown away and some random, but low-frequency, noise that will be included in the reconstruction. \(T_{opt}\) simply represents the point in the observed temporal power spectrum where the magnitude of the noise component is approximately equal to the magnitude of the true signal. It is therefore always advantageous to begin the analysis with \(r(t,\theta)\) containing as little noise as possible. If the noise level is decreased, then higher frequency harmonics may be included in the filtered reconstruction, resulting in better fidelity to the true underlying data. As the noise level increases, the filter cutoff must be moved to a lower frequency, resulting in some degradation of the reconstructed signal. Indeed, this is precisely what Bachrach observed in his error analysis of radionuclide ventriculography time-activity curves.27 For noisy data, \(T_{opt}\) was 3, whereas with high-fidelity data, the optimal reconstruction could include up to the seventh harmonic.

The determination of \(\Theta_{opt}\) by comparison with the
M mode data alone is not entirely satisfactory since the three M mode images at each plane give information about just six endocardial points. Although a higher resolution standard could be obtained from excised pathologic specimens, this might introduce spurious high-frequency data since the fine endocardial trabeculations are smaller than the resolving power of a 3 MHz transducer. A third approach is to use the observed spatial power spectrum to estimate the break point between true signal and noise. From figure 5, this appears to be around the seventh or eighth harmonic, in good agreement with $\Theta_{opt}$ from the M mode data.

**Fourier filtering in overall cardiac image processing.**

Two-dimensional Fourier filtration thus provides a theoretically valid, empirically verified method for reducing the random error in endocardial borders traced from echocardiograms. This method is in no way limited to the echocardiographic technique and could as easily be applied to any space-time description of the heart from contrast ventriculography, radionuclide studies, fast computed tomography, or nuclear magnetic resonance.

The major limitation of the method at present is the need to trace the endocardial borders by hand field by field for an entire cardiac cycle. This is a tedious process that can take up to an hour of an operator’s time. Clearly, before filtered reconstruction can be widely applied, it must be coupled with an efficient scheme for automated edge detection. It is useful, therefore, to consider the place of this technique in an integrated program of computerized echocardiographic processing.

Figure 12 shows such an overall scheme of image analysis. It is divided into two broad segments: (1) low-level processing, concerned with analyzing individual pixels to extract key features (such as endocardial borders, myocardial texture, etc.), and (2) high-level processing, which analyzes these extracted features to yield an anatomic interpretation of the original image. Low-level processing is in many ways analogous to the function of the human retina, whereas high-level processing resembles the integrative function of the occipital cortex.

A recent review described many of the current efforts at low-level processing, including noise reduction in the raw echocardiogram by spatial and temporal averaging, and edge enhancement by gray-scale thresholding, first or second spatial derivatives, or temporal differencing. Even with optimal edge enhancement, most schemes have required some operator input for the final edge extraction to eliminate spurious points and to fill in regions of dropout. With the advent of real-time sequential array processors, however, we may expect increasing sophistication in automated edge detection so that borders may be extracted automatically from an echocardiogram and reduced to the mathematical description, $r(t, \theta)$, at the video field rate.

The current implementation of filtered reconstruction can perform the Fourier transform on $r$, multiply this by the Hanning filter function, and perform the inverse transform to yield $s(t, \theta)$ in under 10 sec on the DEC VAX 11/780. A dedicated array processor would certainly reduce this to a few milliseconds. Once filtered, the reconstructed borders could be used in any of several high-level processing schemes to analyze regional wall motion, such as fractional radial shortening or correlation with previously defined normal ventricles.

An important feature of filtered reconstruction is that it forgives a certain amount of inaccuracy in the preceding steps of edge enhancement and extraction. There is always some uncertainty in the registration of a given cardiac feature onto the video image owing in part to the finite sweep time in forming an image. Thus, no edge detection scheme, however sophisticated, will ever yield results in perfect correlation with the anatomy. Filtered Fourier reconstruction will average these errors over the whole spatiotemporal array and, if these errors have a Gaussian distribution, will reduce the error magnitude to nearly zero.

In summary, we have used spatiotemporal Fourier analysis to demonstrate that traced endocardial borders from echocardiograms contain considerable random error that is predominantly of high frequency. Removal of these high frequencies followed by inverse Fourier transformation has yielded ventricular shapes and motion that are more physiologic than the original data. Comparison with M mode echocardiograms and analysis of Fourier spectral shape both demonstrate that optimal frequency cutoffs for reconstruction are about the fifth temporal and eighth spatial harmonics. Filtered Fourier reconstruction is ideally suited for use in automated echocardiographic analysis systems after endocardial border enhancement and extraction. It will minimize any random error introduced by automated edge detection and allow more accurate subsequent analysis of these borders. The principles of spatiotemporal Fourier filtration are not specific for echocardiography, and should also find application in analysis of ventricular shape and motion by angiography, nuclear magnetic resonance, radionuclide imaging, and computed tomography.
References


Appendix 1: symbols used in text

Lower case letters: Functions and variables in “real” space. Capital letters: Functions and variables in Fourier space.

→: Used to indicate that two functions are related through the Fourier transformation and inverse transform.

\( r(\theta) = R(\theta, \phi) \): Radial distance of centroid to endocardium.

\( t \): Time, with 0 at the peak of the R wave.

\( t_0 \): Temporal sampling interval, 16.67 msec.

\( T \): Temporal frequency, expressed in (R-R interval)^{-1}. For example, \( T = 2 \) corresponds to events that occur twice per cardiac cycle.

\( \phi \): Angular distance around endocardium, 0 between papillary muscles, 360° (2π radian) for full circumference.

\( t_\phi \): Angular sampling interval, 5 degrees.

\( \Theta \): Angular or spatial frequency, expressed in (circumference)^{-1}. For example, \( \Theta = 3 \) corresponds to endocardial deformations occurring three times around the circumference of the endocardium.

\( f(\phi) = \Phi(T, \phi) \): General filter function to remove high frequencies from Fourier transform.

\( \Phi(T, \phi) = S(T, \phi) \): General smoothed version of original r→R transformation.

\( T_c \): Temporal cutoff frequency.
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θc: Spatial (angular) cutoff frequency.

sTcθc(t,θ): Specific smoothed version of r(t,θ) reconstructed with Fourier cutoffs Tc and θc.

s∞,∞(t,θ): Reconstructed r(t,θ) formed with Tc = ∞ and θc = ∞ (thus utilizing all frequencies in R and so identical to the original r).

m(t): Left ventricular internal diameter measured from M mode echocardiogram.

sTcθc(t,θ): Left ventricular internal diameter calculated from reconstructed 2DE along an M line θ' (generally coinciding with m(t)). This is a notational simplification. The actual expression is sTcθc(t,θ') + sTcθc(t,θ' + π), the sum of oppositely directed endocardial rays.

Corr(Tcθc): Correlation coefficient between m(t) and sTcθc(t,θ')

Corr(∞,∞): Correlation coefficient between m(t) and the raw two-dimensional data, r(t,θ') or s∞,∞(t,θ').

Topt: Optimal Fourier filtering cutoffs, defined as yielding the highest value of Corr(Tcθc).

P(T): Power spectrum of ventricular motion.

P(θ): Power spectrum of ventricular shape.

**Appendix 2: Mathematical Principles**

Discrete two-dimensional Fourier transformation. We assume that the distance of the endocardium from the left ventricular centroid (either floating or fixed) may be described by a continuous scalar function of two variables, time and angle about the left ventricular centroid: r(t,θ). Throughout this Appendix, functions with a caret superscript (·) will be continuously defined, whereas the unsuperscripted counterpart will be the same function that has been sampled at discrete intervals. f is periodic in both t, repeating every p seconds (the RR interval), and θ, repeating every 2π radians. To simplify analysis, we normalize t and θ so that they range from 0 to 1 in time periods p and 2π, respectively. When digitized for analysis, f is sampled at intervals t0 and θ0, which are 16.7 msec and 5 degrees, respectively, for the digitizing scheme used in this article. Thus, 72 (360 degrees/5 degrees) rays are sampled for each video field and N = p/0.167 fields are sampled for each cardiac cycle. Using our normalized variables t and θ, t0 = 0.0167/p = 1/N and θ0 = 1/72. The sampled function r(t,θ) is obtained from f by multiplying by a grid of delta functions:

\[ r(t,θ) = r(\text{t}t_0,\text{θ}θ_0) \]

r(t,θ) thus is nonzero only at t = it0 and θ = jθ0, i = 0 to N-1 and j = 0 to 71.

Since f repeats at intervals p and 2π, its Fourier transform is nonzero only for frequencies that are integer multiples of 1/p and 1/2π (1 and 1 in normalized coordinates):

\[ \hat{F}(T,θ) = \hat{F}(1,1) = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(t,θ) e^{-2\pi i (T t + θ θ)} dt dθ \]

\[ = f_0 f_0 \int_{-\pi}^{\pi} f(t,θ) e^{-2\pi i (1 t + 1 θ)} dt dθ \]

which, for the continuous case, is defined for I and J = −∞ to ∞.

We can relate the discrete Fourier transform R(T,θ) to its continuous counterpart, R, by recalling that the Fourier transform of the product of two functions is equivalent to the convolution of the Fourier transforms of the two functions:

\[ (f * g)(T,θ) = \mathcal{F}(f \cdot g)(T,θ) \]

R(T,θ), then, is just R(T,θ) convolved with the Fourier transform of the sampling function δ(t − it0,θ − jθ0), which is similarly a grid of delta functions:

\[ δ(T-It_0,θ-Jθ_0) = δ(T-Nθ-θ-Jθ_0) \]

where I and J range from −∞ to +∞. This grid is spaced every 72 harmonics in T and every N harmonics in θ. The convolution of any function with a delta function simply shifts that function along the axis, e.g., f(x) ∘ δ(x − x0) = f(x − x0).

Thus, R(T,θ) ∘ δ(T − Nθ,θ − θ2) simply replicates R(T,θ) every N temporal harmonics and 72 spatial harmonics summing to form R(T,θ). This frequency shifting is termed aliasing and means that beyond T = N/2 and θ = 36, R(T,θ) includes more and more information from the next copy of R shifted over and therefore is inaccurate.

To summarize in another way: we begin with r(t,θ) sampled at N temporal and 72 spatial points. Its discrete Fourier transform R(T,θ) contains accurate frequency information between θ = −36 to +36 spatial harmonics and T = −N/2 to N/2 temporal harmonics.

**Spatial-temporal power spectrum.** We define the power contained in a given spatiotemporal harmonic as

\[ \mathcal{P}(T,θ) = 1/(72N)^2 \cdot |R(T,θ)|^2 \]

so that total power is equal to the mean squared amplitude of r(t,θ):

\[ \sum_{T} \sum_{θ} \mathcal{P}(T,θ) = 1/72N \sum_{T} \sum_{θ} |r(t,θ)|^2 \]

To examine this power as two functions of time and space (not one function of both), we first center the data [subtract the mean radius from r(t,θ), exactly equivalent to setting R(0,0) = 0]. Then R(T,θ) is summed over T to yield \( P_θ(θ) \) and summed over θ to yield \( P_t(T) \). \( P_θ(θ) \) might be thought of as the power spectrum of ventricular shape as it moves through the cardiac cycle, whereas \( P_t(T) \) represents the power spectrum of motion of each ray around the endocardium. The power of negative frequencies is summed so that \( P_θ(θ) \) is defined for 0 through 36 spatial harmonics, and \( P_t(T) \) is defined for 0 through N/2 temporal harmonics. To adjust for variation in ventricular size, both \( P_θ(θ) \) and \( P_t(T) \) are divided by total power so they are expressed as a percentage of total power contained in a given harmonic. Note also that summing all of the \( P_t(T) \) and \( P_θ(θ) \) yields the same quantity, the total power derived above:

\[ \sum_{T} P_t(T) = \sum_{θ} P_θ(θ) = \sum_{T} \sum_{θ} \mathcal{P}(T,θ) = \text{total power} \]

**Filtered reconstruction.** To illustrate Fourier filtering, consider the univariate example of figure 13. Panel A shows "ideal" data for left ventricular diameter as a function of time; computer-generated random noise is shown in panel C, with a mean of 0 and an SD of 0.2. Their sum in panel E is the sort of noisy signal that might be measured in practice. Its Fourier transform (panel F) is the sum of the low-frequency transform of the true data (panel B) and the lower amplitude but much broader noise spectrum (panel D). If panel F is multiplied by the Hanning filter in panel H to eliminate the higher frequencies, the resultant inverse transformation (panel G) is almost identical to the true data in panel A. Indeed, Monte-Carlo simulation demonstrates that for random Gaussian noise with a flat power spec-

*The convolution f(t) ∘ g(t) is defined by \( \int_{-\pi}^{\pi} f(t)g(t-T) dt \).

†Complex exponential notation is used rather than the more familiar but very cumbersome expansion in sines and cosines, using the relationship \( e^{i\theta} = \cos \theta + i \sin \theta \), where i is the imaginary constant \( \sqrt{-1} \).

*The convolution f(t) ∘ g(t) is defined by \( \int_{-\pi}^{\pi} f(t)g(t-T) dt \).

†Actually, half of these points provide redundant information. Because r(t,θ) is a real function, R(T,θ) = R*(−T,−θ), where * indicates the complex conjugate operator: (x + iy)* = x − iy.
trum out to the Nyquist frequency, $T_{\text{max}}$, applying a filter cutoff of $T_c$ will reduce the magnitude of the noise approximately to $(T_c/T_{\text{max}})^2$ of the original level.

The extension to two dimensions of this filtering notion is straightforward. We postulate that $r(t,\theta)$ is the sum of two functions: the true ventricular signal, $r_v(t,\theta)$, and random noise, $r_{n}(t,\theta)$, distributed with a mean of 0 and an SD of $\sigma_n$. Filtering $R(T,\Theta)$ to eliminate the predominant high-frequency components of $R_n(T,\Theta)$, yet preserving the bulk of the information in $R_v(T,\Theta)$, will yield an improved signal-to-noise ratio on inverse Fourier transformation.

The particular filtering function used in our work is a two-dimensional Hanning (or raised cosine) filter:

$$F_{T_c,\Theta_c}(T,\Theta) = 0.5 \left(1 + \cos\pi\left[(T/T_c)^2 + (\Theta/\Theta_c)^2\right]\right)$$

which is 1 at $T = \Theta = 0$, falls smoothly to 0 on the ellipse (in Fourier space) $(T/T_c)^2 + (\Theta/\Theta_c)^2 = 1$, and is set to zero outside this ellipse. There are several similar filtering functions (Welch, Parzen, Hamming, Blackman), each of which would likely have served well.

It is important to recall that multiplication in Fourier space is equivalent to convolution in real space:

$$(R \ast F) = (r \otimes f)$$

which is the same as a spatiotemporal average of $r$ weighted by $f$. Thus, we require that $F$ have a Fourier transform $f$ that does not have the side lobes of significant magnitude that would interact with distant spatiotemporal points in $r$ (called Gibbs oscillations or “ringing”). For example, the square-wave window:

$$F(T) = 1 \text{ for } |T| < T_c$$

$0 \text{ elsewhere},$ has an inverse Fourier transform

$$f(t) = 2T_c \sin(2\pi T_c t) / 2\pi T_c$$

which has a first sidelobe amplitude that is one-fifth central amplitude, whereas the first sidelobe for the Hanning filter is only about 2% of the center magnitude. Thus, the Hanning filter causes considerably less ringing of the reconstructed signal than would a simple frequency cutoff.
Improved accuracy of echocardiographic endocardial borders by spatiotemporal filtered Fourier reconstruction: description of the method and optimization of filter cutoffs.

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