Calculation of aortic valve area by Doppler echocardiography: a direct application of the continuity equation

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ABSTRACT The continuity equation suggests that a ratio of velocities at two different cardiac valves is inversely proportional to the ratio of cross-sectional areas of the valves. To determine whether a ratio of mitral/aortic valve orifice velocities is useful in determining aortic valve area in patients with aortic stenosis, 10 control subjects and 22 patients with predominant aortic stenosis were examined by Doppler echocardiography. The ratio of (mean diastolic mitral velocity)/(mean systolic aortic velocity), (Vm)/(Va), and the ratio of (mitral diastolic velocity-time integral)/(aortic systolic velocity-time integral), (VTm)/(VTa), were determined from Doppler spectral recordings. Aortic valve area determined at catheterization by the Gorlin equation was the standard of reference. High-quality Doppler recordings were obtained in 30 of 32 subjects (94%). Catheterization documented valve areas of 0.5 to 2.6 (mean 1.1) cm². There was good correlation between Doppler-determined (Vm)/(Va) and Gorlin valve area (r = .90, SEE = 0.23 cm²); a better correlation was noted between (VTm)/(VTa) and Gorlin valve area (r = .93, SEE = 0.18 cm²). The data demonstrate the usefulness of Doppler alone in the determination of aortic valve area in adults with absent or mild aortic or mitral regurgitation and no mitral stenosis. Although the use of mean velocity and velocity-time integral ratios requires accurate measurement of mitral and aortic velocities, it does not require squaring of these velocities or measurement of the cross-sectional area of flow.

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IDENTIFICATION and quantification of aortic stenosis remains a significant problem in adults with systolic murmurs. Cardiac catheterization is frequently required to identify patients with significant aortic stenosis and to quantitate its severity by determining mean systolic pressure gradient and valve area. Doppler echocardiography has proved to be accurate in the identification of patients with aortic stenosis by detecting high velocity1-4 or turbulence5-7 in the aorta downstream from the stenotic valve during systole. Doppler determination of the peak systolic velocity allows calculation of the peak instantaneous pressure gradient by means of the simplified Bernoulli equation.1-2, 8-11 Mean systolic gradient can also be calculated from Doppler velocity recordings by determining the average of multiple systolic instantaneous gradients11-12 or by calculating turbulence variables.13-15

Because pressure gradient is flow dependent, it may be very high in patients in whom aortic valve flow is high and stenosis is only moderate; likewise low gradients may be noted if flow is low despite the presence of severe stenosis. For this reason, aortic valve area is usually estimated from catheterization pressure and flow data by means of the Gorlin equation; the calculated Gorlin valve area is used clinically to judge the severity of valvular stenosis. Aortic valve area can also be calculated noninvasively by combining two-dimensional imaging and Doppler echocardiography.16 Doppler is used to determine the high velocities from the stenotic valve from which the pressure gradient is calculated, as well as to determine the velocities from a normal valve at which cardiac output is measured. Echocardiographic imaging is used to estimate the normal valve cross-sectional area, which is combined with velocity at that site to calculate cardiac output.17 Alternatively, noninvasive Doppler pressure gradient and invasive cardiac output can be used to estimate
valve area.\textsuperscript{18} Echocardiographic imaging alone has not proved to be reliable for estimating aortic valve area.\textsuperscript{19}

The continuity equation provides a simple alternative by which Doppler alone can be used to calculate aortic valve area. The concept is illustrated in figure 1, in which blood flows steadily from left to right through a cylinder with a cross-sectional area of \( A_1 \) at its left end and a cross-sectional area of \( A_2 \) where it narrows. The rate at which blood flows through the large section of the tube can be calculated by multiplying the cross-sectional area of the tube \( A_1 \) by the mean velocity of blood at that site \( (V_1) \). Likewise, the rate of flow across the narrowed segment of the tube can be calculated as the product of cross-sectional area \( A_2 \) times mean velocity \( (V_2) \) within the narrowed segment. The rate at which blood flows into the tube \( (Q_1) \) is the same as the rate at which blood flows out of the tube \( (Q_2) \) because there are no branches in the tube and because flow occurs from left to right only. Mathematically, if \( (Q_1) = (A_1)(V_1) \) and \( (Q_2) = (A_2)(V_2) \), and \( Q_1 = Q_2 \), then:

\[
Q_1 = (A_1)(V_1) = (A_2)(V_2) = Q_2 \quad (1)
\]

If the equation is rearranged, a ratio of cross-sectional areas \( (A_2)/(A_1) \) is shown to be proportional to the inverse ratio of velocities \( (V_1)/(V_2) \):

\[
(A_2)/(A_1) = (V_1)/(V_2) \quad (2)
\]

This model of the circulation can be applied to the heart if we conceptualize the tube as beginning at the normal mitral valve and ending at a stenotic aortic valve. If aortic and mitral valve regurgitation are absent and intracardiac shunts are not present, the volume flow rate through aortic and mitral valves will be equal. If the velocities and cross-sectional areas are relabeled to represent the valve of origin (aortic = a, mitral = m), then equation 2 becomes:

\[
(Aa)/(Am) = (Vm)/(Va) \quad (3)
\]

\[Q_1 = A_1 \quad V_1 = A_2 \quad V_2 = Q_2\]

\[Q_1 = A_1 \quad V_1 = A_2 \quad V_2 = Q_2\]

\[Q_1 = A_1 \quad V_1 = A_2 \quad V_2 = Q_2\]

\[Q_1 = A_1 \quad V_1 = A_2 \quad V_2 = Q_2\]

FIGURE 1. The continuity equation states that in a cylinder in which flow is steady, if input flow rate \( (Q_1) \) equals output flow rate \( (Q_2) \), then cross-sectional area at the entrance \( (A_1) \) times velocity at that site \( (V_1) \) equals cross-sectional area at a second site \( (A_2) \) times the velocity at that site \( (V_2) \).

Thus, if the cross-sectional area of the mitral valve \( (Am) \) is normal and the area of the aortic valve \( (Aa) \) is small because of stenosis, the ratio of mean diastolic velocity across the mitral valve \( (V_m) \) divided by mean systolic velocity across the aortic valve \( (V_a) \) will be small. The reduction in the velocity ratio should be proportional to the reduction in cross-sectional area of the aortic valve.

An alternative approach that combines the continuity equation and concepts important in calculating stroke volume with Doppler is based on the observation that volume/beat \( (Q) \) is proportional to the velocity-time integral \( (VT = \text{area between the velocity curve and zero as shown in figure 2}) \) times the cross-sectional area at the point at which velocity is measured:\textsuperscript{20}

\[
Q = (VT)(A) \quad (4)
\]

If \( (VT) \) is substituted for \( (V) \) in equation 1, then the following relationship is noted:
Q1 = (A1)(VT1) = (A2)(VT2) = Q2  (5)

By rearranging the equation into the form of equation 3, it is suggested that the ratio of valve areas of the aortic and mitral valves will be inversely proportional to the velocity-time integrals at the same valves:

(Aa)/(Am) = (VTm)/(VTa)  (6)

To evaluate the clinical utility of the ratios of mitral/aortic orifice mean velocities and velocity-time integrals to identify patients with aortic stenosis and to quantitate aortic valve area, three hypotheses were investigated: (1) the ratios (Vm)/(Va) and (VTm)/(VTa) vary over a narrow range of values in patients who have no valvular heart disease; (2) the ratios (Vm)/(Va) and (VTm)/(VTa) allow accurate separation of individuals with aortic stenosis from those who do not have aortic stenosis; and (3) the ratios (Vm)/(Va) and (VTm)/(VTa) are directly proportional to aortic valve area in patients with aortic stenosis.

Methods

Two separate patient groups were informed of the risks and benefits of the study and signed consent forms approved by our institutional review board. Group I consisted of 10 patients who had no clinical evidence of valvular heart disease. Group II consisted of 22 consecutive patients who had clinical indications for cardiac catheterization, had no more than mild aortic and/or mitral regurgitation, and were proved to have aortic stenosis by catheterization. Two patients were eliminated from group II because of inadequate Doppler echocardiographic velocity tracings, resulting in a final total of 20 patients.

Cardiac catheterization was performed at rest with the patients in the fasting state. No premedication was given; intravenous meperidine hydrochloride (Demerol) was used to control anxiety or relieve pain. Intravenous atropine and fluids were used if bradycardia or hypotension occurred. Pressure gradients were measured with dual micromanometer catheters passed retrograde across the aortic valve. Cardiac outputs were measured with Fick and/or green dye indicator dilution technique at the time of pressure gradient measurement. Aortic valve area was calculated by the Gorlin equation.21 Left ventricular and aortic root angiograms were obtained in all patients to determine the severity of regurgitation.22 Results of catheterization were not known by the individual analyzing the Doppler data.

Doppler echocardiography was performed with a 2.25 MHz duplex high pulse repetition frequency Doppler ultrasonoscope equipped with split crystal 2.00 MHz continuous-wave Doppler (Advanced Technology Laboratory, Mark 600). Duplex standard and high pulse repetition frequency Doppler were used to detect and quantify mitral valve orifice velocities. The highest mitral valve velocities were recorded by placing the transducer at the cardiac apex and aligning the ultrasound beam and sample volume so as to acquire the highest Doppler frequency shifts. Aortic valve systolic velocities were quantified by continuous-wave Doppler; apical and occasionally right parasternal windows were used to detect the highest Doppler frequency shift. Doppler studies were performed within 48 hr of cardiac catheterization.

Aortic and mitral velocities were calculated with an assumed Doppler angle of zero degrees. All velocity analyses were performed on spectral tracings recorded at paper speeds of 50 or 100 mm/sec with a strip-chart recorder (Advanced Technology Laboratories, Model 130). Aortic valve systolic ejection time (Ts) and mitral valve diastolic flow time (Td) were determined as shown in figure 2. Aortic and mitral velocity-time integrals were calculated by determining the area between the zero baseline and the peak spatial velocities throughout systole and diastole, respectively (figure 2). The aortic velocity-time integral (VTa) was divided by the systolic ejection time (Ts) to determine aortic mean systolic velocity (Va). The mitral velocity-time integral (VTm) was divided by the diastolic flow period (Td) to determine mitral mean diastolic velocity (Vm). All Doppler spectral tracing calculations were made with a digitizing tablet (Summagraphics Model ID-2-CTR-11) and a microcomputer (IBM-PC). All reported measurements represent the average of 3 consecutive beats. Duplicate measurements of Doppler data were made by a single observer and by two independent observers to allow analysis of both intraobserver and interobserver measurement variability.

The ratios of (Vm)/(Va) and (VTm)/(VTa) were regressed against catheterization-determined Gorlin aortic valve areas by means of linear regression analysis. To verify that mean values for (Vm)/(Va) and (VTm)/(VTa) were significantly different in group I vs group II, the paired Student’s t test was used. A one-way analysis of variance for repeated measures was performed to test both interobserver and intraobserver measurement variability. In addition, each measurement was compared with its companion by linear regression analysis to ascertain any systematic differences between observations. All mean values are expressed with standard deviation; p < .05 was considered statistically significant.

Results

At cardiac catheterization, patients from group II had micromanometer-determined mean systolic pressure gradients of 8 to 70 mm Hg (mean 36) and cardiac outputs of 2.9 to 7.5 liters/min (mean = 5.5). Gorlin valve areas ranged from 0.5 to 2.6 cm² (mean 1.1 ± 0.5). Nineteen of the 20 patients had mild aortic and/or mitral regurgitation by angiography. Table 1 summarizes the quantitative catheterization and Doppler echocardiographic findings. The mean values for mitral/aortic mean velocity, (Vm)/(Va), and velocity-time integral ratio, (VTm)/(VTa), were significantly different in groups I and II (p < .01 for both). Both ratios allowed complete separation of individuals in the two groups. As shown in table 1, standard deviations in the control group for (Vm)/(Va) and (VTm)/(VTa) were both small (0.07 and 0.08, respectively).

Figures 3 and 4 display the (Vm)/(Va) and (VTm)/(VTa) ratios determined by Doppler vs the aortic valve area determined by the Gorlin equation at cardiac catheterization. There was a good linear correlation between Gorlin aortic valve area and (Vm)/(Va), (r = .90, SEE = 0.23 cm²). A slightly better correlation with Gorlin valve area was noted when (VTm)/(VTa) was evaluated (r = .93, SEE = 0.18 cm²). It is important to realize that aortic valve area was quantitated accurately by Doppler alone despite the presence of
TABLE 1
Results of cardiac catheterization and Doppler echocardiography

<table>
<thead>
<tr>
<th>Catheterization</th>
<th>Control (n = 10)</th>
<th>Aortic stenosis (n = 20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta P_s ) (mm Hg)</td>
<td>—</td>
<td>8–70 (36)</td>
</tr>
<tr>
<td>CO (l/min)</td>
<td>—</td>
<td>2.9–7.5 (5.5)</td>
</tr>
<tr>
<td>AVA (cm²)</td>
<td>—</td>
<td>0.5–2.6 (1.1)</td>
</tr>
<tr>
<td>Doppler</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| (Vm)/(Va) | 0.333–0.500 (x = .441; s = .07) | 0.100–0.262 (x = .140; s = .05)^
| (VTm)/(VTa) | 0.620–0.855 (x = .733; s = .08) | 0.101–0.440 (x = .210; s = .08)^

\( \Delta P_s \) = peak instantaneous pressure gradients; CO = cardiac output; AVA = aortic valve area; x = mean; s = standard deviation.

*Significant, p < .01.

The latter equation is preferred because of a slightly higher correlation coefficient and slightly smaller standard error of the estimate of 0.009. Similar values were obtained for (Vm)/(Va): mean difference 0.003 (2%), r = .99 and SEE = 0.005. The (VTm)/(VTa) values measured by two different observers were also not statistically different: mean difference 0.011 (5%), r = .99 and SEE = 0.012. Similar results were obtained for (Vm)/(Va): mean difference 0.008 (6%), r = .99 and SEE = 0.008. Thus both the intraobserver and interobserver reproducibility of these measurements were good.

Discussion

Current Doppler echocardiographic techniques allow accurate noninvasive determination of the peak instantaneous pressure gradients across stenotic aortic valves in most adults.1-4, 9-14 Although this is extremely helpful, Doppler pressure gradients alone have not eliminated the need for hemodynamic evaluation in patients with aortic stenosis because (1) pressure gradients are flow dependent and do not always reveal the

FIGURE 3. The ratio of Doppler-determined mean diastolic mitral velocity divided by mean systolic aortic velocity (Vm)/(Va) correlates well with aortic valve areas determined by the Gorlin equation at catheterization. Valve areas are in square centimeters and can be calculated from Doppler data by the regression equation shown.

FIGURE 4. The ratio of Doppler-determined mitral diastolic velocity-time integral divided by aortic systolic velocity-time integral (VTm)/(VTa) correlates well with aortic valve areas determined by the Gorlin equation at catheterization. Valve areas are in square centimeters and can be calculated from Doppler data by the regression equation shown.
severity of aortic stenosis; (2) pressure gradients obtained at noninvasive study frequently differ from those determined at cardiac catheterization because of differences in hemodynamics at the times of the nonsimultaneous catheterization and Doppler studies; (3) the peak instantaneous pressure gradient calculated by the simple Bernoulli equation (∆P = 4 V^2) is not the same pressure gradient determined at catheterization (peak instantaneous vs peak-to-peak or mean systolic gradient); and (4) Doppler pressure gradients can seriously underestimate the actual gradient if the Doppler angle is more than 30 degrees because the error in velocity measurement is squared.

The continuity equation provides a method by which aortic valve area, a measure commonly derived at catheterization, can be calculated directly from Doppler velocity recordings. Determination of valve area, rather than pressure gradient, has the advantage of providing an index of severity of aortic stenosis that is theoretically independent of changes in valve flow and thus should be more stable under the different hemodynamic conditions encountered in the invasive vs noninvasive laboratories. If the equations provided are used, aortic valve area is calculated in square centimeters, the same unit used for reporting valve area at catheterization.

As with other Doppler techniques that quantitate velocity, the transducer must be aligned parallel to the velocity vectors from the valve. Failure to do so leads to underestimation of true valve orifice velocity if the Doppler angle is assumed to be zero. When the mitral/aortic velocity or velocity-time integral ratios are used to calculate aortic valve areas, underestimation of mitral velocity results in underestimation of true valve area; underestimation of aortic velocity results in overestimation of valve area. The error induced in valve area calculations by a larger-than-anticipated Doppler angle is linear. The linear error induced in the continuity equation by underestimation of true velocity is smaller than the error that would be anticipated in a pressure gradient calculated from the Bernoulli equation, in which the same velocity is squared.

Alternative echocardiographic techniques that use the Gorlin equation to determine aortic valve area require calculation of both cardiac output and stenotic valve pressure gradient. If the mitral valve is used as the site of volume flow measurement, it is assumed that mitral and aortic valve flows are equal; thus, mitral and aortic regurgitation must be small or absent. Two Doppler velocity measurements (aortic and mitral valve orifice) and two imaging measurements (two dimensional and M mode determination of mitral valve cross-sectional area) are required. Although the pressure gradient information is accurate, measurement of the average diastolic mitral valve cross-sectional area is frequently difficult in adults. The alternative approach of combining Doppler aortic valve pressure gradients with cardiac output determined by thermodilution is accurate but requires right heart catheterization and is not suitable for screening large numbers of patients with systolic murmurs. The technique we propose requires only two velocity measurements; imaging is not necessary. In addition, it is totally noninvasive and thus avoids the need for right heart catheterization to determine cardiac output.

Direct application of the continuity equation to determine valve area requires that the flow rate across both the stenotic valve and the valve used to derive the velocity ratio (the index valve) be equal. Thus valve regurgitation involving either valve must be mild or absent. Furthermore, the index valve must be free of stenosis. Fortunately, thorough Doppler examination can accurately identify regurgitation or stenosis of the index valve and regurgitation of the stenotic valve as well as exclude the presence of intracardiac shunts. Doppler also allows identification of patients with moderate and severe regurgitation who should be evaluated by other techniques. Further research is required to determine whether the tricuspid and pulmonary valves will serve as alternative index valve sites when mitral regurgitation is moderate or severe.

The concepts presented illustrate the application of the continuity equation to the clinical problem of quantifying aortic valve area echocardiographically. The approach requires determination of simple variables (velocity-time integrals or mean temporal velocities) from Doppler alone. Although these Doppler estimates of valve area have not been compared with direct determination of aortic valve area at surgery, they correlate well with a clinically useful standard of reference, the Gorlin valve area. Interestingly, the greatest differences between Doppler and Gorlin valve areas in our study were noted in individuals with Gorlin aortic valve areas below 1.5 cm^2, a population in which Gorlin valve areas may be inaccurate. In conclusion, this study demonstrates a simple method for determining aortic valve area in which accurate measurement of aortic and mitral valve orifice velocities is required, but in which neither squaring of velocities nor measurement of mean flow cross-sectional areas is required.

The members of the cardiac catheterization research team who were instrumental in this project include David Foster,

References
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