Echocardiographic measurement of right ventricular volume

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ABSTRACT The volume of the right ventricle can be determined angiographically from its projections in two mutually perpendicular planes. Echocardiographic techniques for measuring right ventricular volume, however, have been more difficult and less successful. In this study, a method was developed for calculating right ventricular volume from two intersecting cross-sectional echocardiographic views: the apical four-chamber and subcostal right ventricular outflow tract views. First, the areas and lengths of casts of 12 human right ventricles obtained at autopsy were directly measured in the chosen views. Actual cast volumes correlated best with a formula giving volume as \( \frac{2}{3} \) times the area in one view times the long axis in the other view. The degree of correlation was similarly high for calculations involving the area derived from either view and the length of the roughly orthogonal section. This relationship for right ventricular volume was then confirmed with two-dimensional echocardiographic images of hollow latex molds made from the casts \((r = .95, p < .0001)\). The significance of these findings is discussed in relation to angiographic results and models of the right ventricle.


THE DETERMINATION of right ventricular volume is conceptually more difficult than that of left ventricular volume. While the left ventricle resembles the ellipsoid, a convenient volumetric model, the right ventricle has a complex and crescentic shape, defying simple geometrical description. The problem is compounded by irregular trabeculations, a separate infundibulum, and variations in right ventricular shape with altered loading conditions.

Despite these difficulties, many angiographic studies have shown that right ventricular volume can be accurately assessed from the projections of that ventricle in two perpendicular planes. The craniocaudal length of the ventricle is the common long axis of both projections. The frontal and lateral, or 30 degree right anterior oblique and 60 degree left anterior oblique, views are the ones generally used. Simple combinations of area and length in these projections can estimate volume at least as well as the more cumbersome Simpson’s rule techniques.1–12

Attempts to determine right ventricular volume echocardiographically have been made relatively recently. The substernal right ventricle is less accessible than the left ventricle and its dimensions more difficult to standardize. Despite the advent of cross-sectional scanning, obtaining the set of two perpendicular views used in angiography — particularly a frontal or right anterior oblique equivalent — has been difficult. To date, limited results have been obtained with single and biplane techniques.13–18

The purpose of this study was to determine whether right ventricular volume could be assessed with a simple combination of areas and lengths from two cross-sectional echocardiographic views that are readily available. The two planes chosen were the apical four-chamber and subcostal right ventricular outflow tract views, which intersect and contain information regarding the right ventricular apex as well as the inflow and outflow tracts.19, 20 Initially, postmortem casts of human right ventricles were directly measured in these two planes to determine a relationship for volume.21 Before applying this formula to echocardiographic views of the same objects, hollow latex molds were made from the casts to provide more discrete echogenic interfaces and to circumvent one of the problems of studying solid casts: distortion of the echocardiographic image by variations in the speed of sound passing through different thicknesses of rubber in the cast. Finally, the formula obtained for right ventricular
A

B

FIGURE 1. Frontal view of a right ventricular cast (A) and latex mold (B). The right ventricular apex is at the right lower corner of each panel. The tricuspid annular plane is viewed en face in the lower left portion of each panel; the outflow tract appears in the left upper corner.

The volume was applied to echocardiographic views of the latex molds.

Methods

Materials. Intact human hearts of various sizes were obtained at autopsy from 12 adult patients with and without heart disease. A silicone disk fitted with an injection cannula was sutured into the tricuspid valve orifice, and the pulmonary artery was clamped just above the pulmonic valve. With the heart suspended in normal saline, silicone rubber was injected through the cannula at a pressure of 10 cm water. After the vulcanized casts were cleaned and dried, their volumes were measured by water displacement.

Each cast was suspended from a stand and coated with a 1 to 2 mm layer of latex mold-making rubber. The hardened molds were cut along one face and the casts removed. Continuity of the mold surface was restored with sutures and rubber cement (figure 1).

Measurements. The casts were directly measured in two planes: (1) the apical four-chamber plane, passing through the right ventricular apex and the center of the tricuspid valve and perpendicular to the interventricular septum and (2) the right ventricular outflow tract plane, passing through the pulmonic valve parallel to the tricuspid valve orifice and perpendicular to the interventricular septum. The casts were sectioned along a given plane to provide direct measurements of areas and longest lengths and then reassembled to allow study of the other plane (figure 3).

FIGURE 2. Echocardiographic views comparable to the planes used in vitro in this study. (A), Apical four-chamber and (B) right ventricular outflow tract. Line drawings appear in the right-hand panels. LA = left atrium; LV = left ventricle; MV = mitral valve; PA = pulmonary artery; RA = right atrium; RV = right ventricle; TV = tricuspid valve.

The hollow latex molds were immersed in a water bath and completely filled with water through a small hole. The two cross-sectional views defined above were obtained with the 5 MHz rotating beam transducer of an Advanced Technology Laboratories 850A scanner. The plane of the ultrasound beam was rotated and angulated to avoid obliquity of the views and to maximize clarity.

The inner echocardiographic borders in a given view were planimetered with a Microsonics computer to obtain the cross-
sectional area in that view. In zones of echocardiographic dropout, the nearest two visualized points were connected by a straight line. The maximum length (L) in a given view was taken as the apex-to-base height for triangular images; if a planar image was not clearly triangular, its longest axis was used (figure 4). Five to seven measurements of each area and length were taken and averaged. For each set of repeated measurements in a given view, the SD was, on the average, 5.1% of the mean.

Statistical analysis. Calculated and measured volumes were correlated by the method of linear regression. The SEE for right ventricular volume assessment by echocardiography was obtained by an analysis of variance and covariance with repeated measures (BMDP2V statistical package, UCLA, 1981).

Results

Since area and length measurements in two views were available, an equation of the form \( V = cA_1L_2 \) was chosen to represent right ventricular volume (V), where \( c \) is a constant, \( A_1 \) is the area in one viewing plane, and \( L_2 \) is the long-axis length in the other plane. \( V(\text{AP}) \) was defined as the volume calculated from the area in the apical four-chamber view and the length in the outflow tract view. Similarly, the area in the outflow tract view and the length in the apical view were used to calculate \( V(\text{OT}) \).

**FIGURE 4.** Echocardiographic views of the latex molds in the apical four-chamber (A) and right ventricular outflow tract (B) views (left-hand panels). The center panels show the areas outlined for planimetry in each view, with long-axis lengths (L) demonstrated in the right-hand panels.

Direct measurements of casts. To fit the data, the constant was chosen as a ratio of small integers. Excellent correlations for both \( V(\text{AP}) \) and \( V(\text{OT}) \) were obtained with \( c = 2/3 \), which also gave a slope of 1.0 for the line relating \( V(\text{AP}) \) and measured cast volume (table 1; figure 5).

**Echocardiographic measurements.** \( V(\text{AP}) \) and \( V(\text{OT}) \) were calculated with the echocardiographic measurements and \( c = 2/3 \) as determined from the cast studies (table 2). Linear regression analysis showed that each of these echocardiographically derived volume estimates correlated well with \( V_m \), the measured cast volume, as follows (all volumes are in ml): \( V(\text{AP}) = 1.1V_m - 3.9 \), with \( r = .946 \) (p < .0001) and an SEE of 8.3; and \( V(\text{OT}) = 1.02V_m - 2.1 \), with \( r = .945 \) (p < .0001) and SEE = 7.8 (figure 6).

In addition, the two volume estimates from the echocardiographic data correlated well with one another: \( V(\text{OT}) = .90V(\text{AP}) + 3.3 \), with \( r = .962 \) (p < .0001).

The repeated-measures analysis of variance and covariance for echocardiographically derived \( V(\text{AP}) \), \( V(\text{OT}) \), and \( V_m \) gave an SEE of 5.2 ml.

**TABLE 1**

<table>
<thead>
<tr>
<th>Volume formula</th>
<th>Regression equation</th>
<th>( r )</th>
<th>SEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A(\text{OT}) \times L(\text{AP})/2 )</td>
<td>( y = 8.9 + 0.66x )</td>
<td>.91</td>
<td>6.7</td>
</tr>
<tr>
<td>( 2A(\text{OT}) \times L(\text{AP})/3 )</td>
<td>( y = 11.9 + 0.88x )</td>
<td>.91</td>
<td>9.0</td>
</tr>
<tr>
<td>( A(\text{AP}) \times L(\text{OT})/2 )</td>
<td>( y = 3.4 + 0.78x )</td>
<td>.94</td>
<td>6.2</td>
</tr>
<tr>
<td>( 2A(\text{AP}) \times L(\text{OT})/3 )</td>
<td>( y = 4.6 + 1.0x )</td>
<td>.94</td>
<td>8.3</td>
</tr>
</tbody>
</table>

\( A(\text{AP}) \) = area in the apical four-chamber plane; \( A(\text{OT}) \) = area in the right ventricular outflow tract plane; \( L(\text{AP}) \) = length in the apical four-chamber plane; \( L(\text{OT}) \) = length in the outflow tract plane; \( y \) = calculated volume in ml; \( x \) = measured cast volume.
ventricular cast volumes. To obtain right ventricular volume, they summed cylindrical volume elements, the borders of which were defined by the projections of the right ventricle in the frontal and lateral planes. Others have shown that triangular or elliptical volume elements give similar results. In vivo, right ventricular stroke volume obtained in this manner correlates with stroke volume measured with a flowmeter in dogs; it also correlates with right ventricular stroke volume obtained by measuring left ventricular stroke volume angiographically and assuming equality in the steady state. In general, however, these calculations overestimate right ventricular volume by as much as 40% and are cumbersome, requiring a computer for their application.

Despite the complexity of right ventricular structure, Arcilla et al. found that its volume could be assessed equally well by a calculation simpler than that required by Simpson's rule. The ventricle was project-

<table>
<thead>
<tr>
<th>Cast No.</th>
<th>V(AP)</th>
<th>V(OT)</th>
<th>V_m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.4</td>
<td>18.6</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>27.6</td>
<td>26.7</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>26.0</td>
<td>25.2</td>
<td>29</td>
</tr>
<tr>
<td>4</td>
<td>27.7</td>
<td>28.3</td>
<td>32</td>
</tr>
<tr>
<td>5</td>
<td>52.6</td>
<td>49.2</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>47.8</td>
<td>50.3</td>
<td>45</td>
</tr>
<tr>
<td>7</td>
<td>37.7</td>
<td>34.4</td>
<td>50</td>
</tr>
<tr>
<td>8</td>
<td>56.8</td>
<td>62.8</td>
<td>54</td>
</tr>
<tr>
<td>9</td>
<td>58.8</td>
<td>62.1</td>
<td>57</td>
</tr>
<tr>
<td>10</td>
<td>59.9</td>
<td>50.7</td>
<td>60</td>
</tr>
<tr>
<td>11</td>
<td>100.1</td>
<td>81.9</td>
<td>82</td>
</tr>
<tr>
<td>12</td>
<td>84.3</td>
<td>88.6</td>
<td>90</td>
</tr>
</tbody>
</table>

V(AP), V(OT), and V_m are as defined in the text.
ed in two perpendicular planes, intersecting along a common axis that was generally the cranio-caudal long axis of the ventricle. Volume was then calculated as \( V = c \times A_1 \times A_2 / L_{\text{common}} \), where \( c \) is a constant, \( A_1 \) and \( A_2 \) are the areas of the two projected silhouettes, and \( L_{\text{common}} \) is the length of the long common axis. This formula was suggested by the construction of a parallelepiped equivalent to the ventricle, with sides equal in area to those of the two right ventricular projections. The same formula has been subsequently derived from other models — those incorporating biplane area-length techniques (by analogy to the left ventricular ellipsoid), the triangular prism, and the pyramid; the proportionality constant is ultimately empirical.3, 4, 8, 9

The planes used in most studies are the frontal and lateral, or 30 degree right and 60 degree left anterior oblique, views. Lange et al.10 have summarized the planes used and formulas obtained. For biplane multiple-slice or area-length techniques, the precise set of planes used makes only a small difference in the proportionality constant.10, 11 Single-plane techniques, however, are more critically dependent on the orientation of imaging planes with respect to intrinsic right ventricular axes, and assume a constant relationship between the dimensions of the ventricle in two projections.3, 12

Echocardiographic techniques. The application of cross-sectional echocardiography to the problem of right ventricular volume assessment has been relatively recent. Despite the advent of two-dimensional techniques, it has been difficult to obtain readily standardizable views of the right ventricle that furnish a complete circumference.19, 20 The set of two perpendicular views so useful in angiography, and particularly a frontal or 30 degree right anterior oblique equivalent, has been difficult to obtain.

Bommer et al.13 showed that volumes of human right ventricular casts correlated well with a linear combination of cross-sectional area and maximum short-axis length in the apical four-chamber view. However, they made no claim that this index, computed with information derived from only one plane, actually represented ventricular volume.

Only Saito et al.14 working with children 6 months to 6 years of age have reported a subcostal frontal-view approximation. Precise borders of the right ventricular apex were often difficult to obtain in this view, which has not been reported in other patient groups. They obtained the outlines of the ventricle in this view and in a roughly orthogonal lateral view similar to that of the outflow tract used in the current study. Application of Simpson’s rule or a biplane area-length method to these two outlines provided only moderately good correlations (\( r = .85 \)) with biplane angiographically determined volumes.

Ninomiya et al.15 obtained right ventricular volume by Simpson’s rule with the apical four-chamber and parasternal short-axis planes in patients after Mustard repair. Although right ventricular ejection fraction correlated well with that from biplane angiography, volumes did not. The two views studied in this limited population are not necessarily perpendicular, and do not include information about the infundibulum. Furthermore, the short-axis view may be difficult to standardize with respect to internal right ventricular references.

Watanabe et al.16 were able to assess the volume of the right ventricular body — that is, all segments except the outflow tract — by applying Simpson’s rule to the borders of the ventricle in two perpendicular apical views. Correlation with angiographic ventricular body volume was better for end-diastole than for end-systole. However, to obtain the apical view perpendicular to the standard four-chamber one, they were sometimes forced to use a specially constructed table to provide the required transducer access to the apex of a prone patient. This technique excludes the right ventricular outflow tract and, in addition, requires a Simpson’s rule calculation. Furthermore, echocardiographic estimates of ventricular body volume were only about half of the angiographic ones.

Since the initiation of our study, Starling et al.17 have reported moderately good correlations between radionuclide counts in the right ventricle and echocardiographic volumes obtained by a relatively simple calculation. Two subcostal views were used: the right ventricular outflow tract view and an intersecting inflow tract one similar, but not identical, to the apical four-chamber view. Right ventricular volume was calculated as proportional to \( A_{\text{inflow}} \times L_{\text{outflow}} \), as suggested by a pyramidal model. Unfortunately, both views could be obtained in only 64% of the patients studied; the inflow tract view was particularly difficult to obtain. No angiographic correlation was attempted, so the proportionality constant between volume and \( A \times L \) could not be determined. The form of the results of Starling et al., however, is in consonance with that of our findings.

To circumvent the need for two echocardiographic views, Hiraishi et al.18 used primary echocardiographic data from the apical four-chamber view only, generating equivalent frontal and lateral view dimensions by correlation with biplane angiograms in the

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same patients. The problem was thereby reduced to the angiographic one previously studied. Using formulas of the form \( V = c \times A_1 \times A_2 / L_{\text{common}} \), as defined above, they obtained good correlations for right ventricular end-diastolic volume and ejection fraction compared with biplane angiographic results. This technique derives primary information from one plane only, possibly explaining the need to modify the formula in the case of tetralogy of Fallot. In addition, angiographic results are correlated with a combination of echocardiographic and angiographic data, rather than with echocardiographic data alone.

The current study demonstrates that the volume of human right ventricular casts can be accurately assessed with the use of one area and one length from each of two intersecting views, namely the apical four-chamber and right ventricular outflow tract views defined previously. When \( A_1 \) is the cross-sectional area in one view and \( L_2 \) is the long axis of the intersecting view, right ventricular volume is \( 2 A_1 L_2 / 3 \), with minor adjustments to fit the actual data. The values used in this formula are obtained entirely echocardiographically.

It is obtained from two planes that encompass all segments of the ventricle, including the infundibulum, so that no assumptions are made relating these two planes. The views in this study should be obtainable in most routine echocardiographic studies.

It is not surprising that an equation of this form works; it is in consonance with the angiographic formula \( V = c A_1 A_2 / L_{\text{common}} \), as defined above. This last formula describes the volume of a parallelepiped equivalent to the right ventricle. Consider \( L_2 \) to be the dimension of the parallelepiped in a plane perpendicular to that containing \( A_1 \). Then \( A_2 = c' L_2 \times L_{\text{common}} \), so that \( V \) is proportional to \( A_1 L_2 \).

Although the angiographic formula of Ferlinz et al. was motivated by the theoretical model of a pyramid of volume \((1/3) A_1 L_2 / 2\), the factor \(2/3\) obtained in our study does not conflict with their results. The actual formula used to fit their data was \( V = 2 A_1 A_2 / 3 L_{\text{common}} \). The correlation of volume with the results obtained with such an equation does not prove that the ventricle has a pyramidal form, since similar formulas for volume are predicted by a variety of other models as well. Furthermore, the planes containing \( A_1 \) and \( L_2 \) are perpendicular in the angiographic derivation, whereas they need not be so related when the methods from our study are used.

It is of interest to rationalize the formula obtained in this study by relating it to the volume of a figure resembling human right ventricular casts. One desirable feature of such a model would be a flattened, crescentic shape. In addition, the figure should appear to taper in a pyramidal fashion when viewed from either the apex or from the vicinity of the pulmonic valve. The similarity of the results of the \( V(AP) \) and \( V(OT) \) calculations in this study also corresponds to this basic symmetry in two intersecting views.

Although \( V = 2 A L_2 / 3 \) describes the volume of the prolate ellipsoid shown in figure 7, \( A_1 \) such a structure does not truly resemble the right ventricle. For this formula to hold, for example, one of the areas used in this study would have to be moved back to the middle of the ventricle, as diagrammed. Any portion of a half-cylinder, half-cone cut along a plane parallel to its long axis will have a volume equal to \( 2 A L_2 / 3 \) (figure 7, B). Such a figure resembles the tapering, crescentic right ventricle, and can be cut along any chord of the circular base to give a relatively flat structure. The same result is obtained when similarly tapering structures with rectangular, trapezoidal, or triangular bases are used (figures 7, C through E). Unlike the actual right ventricular casts, however, these structures taper only in one direction and not in a perpendicular one, although the prism-pyramid approaches the form of the right ventricle.

While a pyramid can be constructed to taper in two perpendicular directions, its volume is only \( A L_2 / 3 \). Results of inspection of human right ventricular casts, however, suggest that the right ventricle has a body giving it a volume larger than that of a simple pyramid. This body segment of the right ventricle can be viewed as beginning at the tricuspid valve orifice and extending a variable distance toward the apex. A model that takes this segment of the ventricle into account can be constructed with the use of a rectangular solid corresponding to the body, topped and flanked by four tapering pyramidal structures (figure 7, F). This model is a flexible one; that is, no initial restrictions are placed on the relative dimensions of its various segments. For the purpose of the rationalization, the two perpendicular faces of this model are considered to be roughly equivalent to the apical four-chamber and right ventricular outflow tract views. Setting the volume of this structure to \( 2 A L_2 / 3 \) puts little restriction on the dimensions of the model, as discussed in the Appendix. The requirement that \( V = 2 A L_2 / 3 \) regardless of which face is used to obtain the area adds an additional restriction. A similar model (figure 7, G) with only two tapering structures corresponding to the apex and the outflow tract, respectively, is more stringently restricted, in that assigning a value to one of its geometrical parameters determines the value of all the others.
DIAGNOSTIC METHODS–ECHOCARDIOGRAPHY

The purpose of these models is not to provide a quantitative fit of the right ventricle. Rather, they serve to make the formula obtained for volume seem reasonable. We have demonstrated that a geometric structure can be constructed resembling the right ventricle with respect to its overall form and its body segment, and that such a structure can have a volume of 2 A L/3 without unreasonable restriction of its dimensions. Qualitatively, the factor \( \gamma/3 \) can be viewed as expressing the fact that the right ventricle has a body segment giving it a volume greater than that of a simple pyramid. Since the formulation incorporates a body, one might expect it to hold even when that segment of the ventricle becomes increasingly prominent, as in right ventricular dilatation.

Limitations. Despite the high correlation coefficient, individual calculated volumes may differ from actual ones, particularly with more unusual cast shapes, as in the case of cast 7. Areas of echocardiographic dropout, especially from surfaces that are nearly parallel to the imaging beam, can generate error. The lack of internal right ventricular references can lead to incorrect plane positioning; this might be less of a problem in patients than in casts.

The regression equation obtained for V(AP) vs measured volume from echocardiographic data is similar to that obtained for V(AP) from direct cast measurements. The equations for the V(OT) calculations are somewhat different, however, suggesting either that the primary data for that calculation in the casts are different from those in the molds or that the V(AP) calculation is the more consistent of the two volume assessments. In addition V(AP), although correlating well with V(OT), need not equal it for any given cast; this is, in fact, the case.

Potential problems in the application of this technique to real-time studies are similar to those pertaining to any ultrasound method. The required planes can probably be obtained and standardized more easily than the ones used in previous approaches. Endocardial definition and visualization of the anterior wall of the right ventricle are universal problems. It remains to be seen whether this formula for volume varies with the phase of the cardiac cycle, altered loading conditions, and different bodily positions. At this time, right ventricular volumes cannot be measured directly for correlations in vivo. Future work could involve comparing biplane angiographic volumes and stroke volumes in patients, possibly studied with digital subtraction techniques, with those calculated with the current echocardiographic approach.

In summary, therefore, a simple product of a cross-sectional area and an intersecting length has been found to correlate with human right ventricular cast volumes. Although three-dimensional reconstruction of the right ventricle may ultimately be possible, it will require the imaging of multiple planes, while our technique will retain the advantage of relative simplic-
ity. Finally, the form of the equation obtained is qualitatively reasonable and provides insight into the shape of the right ventricle.

We thank Mr. John Newell for his assistance with the statistical analysis.

References

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Appendix

Figure 7. F illustrates a model comprising a rectangular parallelepiped body and four pyramidal structures. It has two mutually perpendicular faces of areas \( A_1 \) and \( A_2 \), and lengths \( L_1 \) and \( L_2 \), respectively. All segments of each face lie in the same plane. The fraction of each length \( L_0 \) occupied by the rectangular portion of the structure is denoted by the corresponding \( b \). \( D \) is the width of the structure; \( d \) is the fraction of this width that is occupied by the rectangular portion. \( a \) is the fraction of area \( A_1 \) contributed by the rectangular body, \( a_1 \) is the fraction contributed by the outflow tract pyramid, and \( a_2 \) is the contribution of the flanking pyramids, so that

\[
a_1 + a_2 + a_3 = 1 \quad (1)
\]

The volume of the rectangular portion \( = (a_1 A_1) (b_2 L_2) \), that of the outflow tract pyramid is \( 2 \{ (a_2 A_2) (b_2 L_2) / 3 \} \), that of the apical pyramid is \( (a_1 A_1) (L_1^2) / 3 \), and that of the flanking pyramids combined is \( (a_3 A_3) (b_2 L_2) / 2 \). If we add all these partial volumes and set the result equal to \( 2 A_1 L_2 / 3 \), with equation \( 1 \) we obtain

\[
b_2 = (4 - 2a_1b_1)(4 - a_1) \quad (2)
\]

Since \( b_2 \leq 1 \)

\[
a_1 \geq a_1/2 \quad (3)
\]

Equations 1 and 3 imply

\[
a_3 \leq 1 - (3a_1/2) \quad (4)
\]

Solving equations 1 and 2 for \( a_2 \) and requiring that it be \( \geq 0 \) gives

\[
b_2 \geq (2 + 2a_1)/4 - a_1) \quad (5)
\]

Applying \( b_2 \leq 1 \) to equation 5 implies that

\[
a_1 \leq 2/3 \quad (6)
\]

and that

\[
b_2 \geq 1/2 \quad (7)
\]

Equations 3 through 7, then, describe ranges for \( a_1, a_2, \) and \( b_2 \) as functions of \( a_1 \); these relations are plotted in figure 8. Similar results for \( b_1 \) and the fractions of area \( A_1 \) hold by symmetry.

Several qualitative results are apparent. First, the rectangular body must occupy at least half the length of each side. This requirement can be compared with that in figure 7, C through E, in which structures must taper along half their lengths to have a factor of 1/2 in their volume formulas. Second, equation 2 implies that as \( a_1 \), the area fraction of one side of the body segment, increases, \( b_2 \), the length fraction in a perpendicular plane, decreases. Finally, \( a_1 \) is usually very small, so that the allowable ranges of \( a_1, a_2, \) and \( b_2 \) are generally large.

To relate \( b_1, b_2, \) and \( d \), we expand \( A_1 \) and \( V \) in terms of those variables, and then equate \( V \) to \( 2 A_1 L_2 / 3 \).

\[
A_1 = d D b_1 L_1 + (d D [1 - b_1]) L_1 / 2 + (D b_1 L_1 [1 - d / 2]) = d L_1 (b_1 + d) / 2 \quad (8)
\]

The method of expansion for \( V \) is similar to that used in obtaining equation 2. We get

\[
b_2 = (4d + 4b_1 [1 - d]) / (4d + b_1 [3 + d]) \quad (9)
\]

Since \( b_2 \leq 1 \),

\[
d \geq 1/5 \quad (10)
\]
**MODEL PARAMETERS**

![Model Parameters Diagram](image)

**FIGURE 8.** Relationships between parameters of the flexible model (figure 7, F). \( a_1 \) is the fraction of area \( A_1 \) contributed by the rectangular body; \( a_2 \) is the fraction of \( A_1 \) contributed by the outflow tract; \( a_3 \) is the fraction of \( A_1 \) contributed by the flanking structures; \( b_1, b_2, \) and \( d \) are as defined in figure 7.

Solving equation 9 for \( b_1 \) and requiring that \( b_1 \geq 0 \) gives

\[
b_2 > \frac{4 (1 - d)}{3 + d}, \text{ or } b_2 = 1 \quad (11)
\]

Equations 9 through 11 describe the relationships between \( b_1, b_2, \) and \( d, \) and are plotted in figure 8.

Qualitatively, as the length fraction of the body segment in one plane increases, the length fraction in the other plane decreases.

If we require that \( V = 2 A_1 L_2/3 = 2 A_2 L_1/3 \) then, by equation 8 and the corresponding equation for \( A_2 \),

\[
b_1 = b_2 \quad (12)
\]

That is, this symmetry requires equal length fractions for the rectangular body in both planes. The values of the common \( b \) can be obtained from the points of intersection of the line \( b_1 = b_2 \) and any given \( b_2 = b_1 - \text{vs} - b_1 \) curve (a function of \( d \)), as shown in the lower right hand panel of figure 8.

Figure 7, \( G \) illustrates a simpler model with only two pyramidal components; definitions are similar. Repeating the initial steps of the above analysis yields

\[
a_1 = 2 (1 - b_2) \quad (13)
\]

\[
a_2 = 2b_2 - 1 \quad (14)
\]

and

\[
b_2 \geq 1/2 \quad (15)
\]

Similar equations for \( b_1 \) and the fractions of area \( A_3 \) hold by symmetry. Therefore, as the area fraction of the body in one plane increases, its length fraction in the other plane decreases.

In addition, choosing \( a_1, a_2, \) or \( b_2 \) determines the other two in this model.

Figure 7, \( G \) is equivalent to figure 7, \( F \) with \( d = 1 \); therefore, by equation 9

\[
b_2 = 1/(1 + b_1) \quad (16)
\]

Requiring that \( V = 2 A_1 L_2/3 = 2 A_2 L_1/3 \) implies, as before, that \( b_2 = b_1 \). Finally, applying this equality to equation 16 gives \( b_1 = b_2 = 0.618 \).

Thus, mathematical models qualitatively resembling the right ventricle can be constructed without unreasonable restriction — that is, their parameters can vary over a wide enough range so that variation in ventricular shape and size is not precluded.
Echocardiographic measurement of right ventricular volume.
R A Levine, T C Gibson, T Aretz, L D Gillam, D E Guyer, M E King and A E Weyman

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