Assessment of Left Ventricular Stiffness in Primary Myocardial Disease and Coronary Artery Disease

By Israel Mirsky, Ph.D., Peter F. Cohn, M.D., Jay A. Levine, M.D., Richard Gorlin, M.D., Michael V. Herman, M.D., Thomas H. Kreulen, M.D., and Edmund H. Sonnenblick, M.D.

SUMMARY
Stress-strain relations (σ-ε) were obtained in the form dσ/de = kσ + c, where k is a stiffness constant. Utilizing the pressure-volume relation dP/dV = αP, the elastic stiffness (dσ/de) and k were evaluated at end diastole in 10 patients with normal ventricles (N), in 34 patients with coronary artery disease (CAD), and in 22 patients with primary myocardial disease. This latter group was classified into Type I (normal contraction patterns and elevated end-diastolic pressure), Type II (hypertrophy without obstruction), and Type III (hypokinetic and/or asynergic).

The mean values and standard error of the means (SEM) of k and elastic stiffness were 14.8 ± 0.7, 329 ± 54 gm/cm² (N); 17.8 ± 0.3, 684 ± 80 gm/cm² (CAD); 18.2 ± 0.5, 1133 ± 127 gm/cm² (Type I); 22.4 ± 1.2, 856 ± 150 gm/cm² (Type II); 18.7 ± 0.6, 1629 ± 948 gm/cm² (Type III).

These studies indicate that dP/dV, wall stress, and volume-mass ratio are the important determinants of stiffness. 1) Normal stiffness levels can be recorded from hypertrophied ventricles, 2) CAD patients with end-diastolic pressure ≤ 12 mm Hg have normal stiffness levels, 4) normal contraction patterns and normal stiffness levels are not necessarily related.

Additional Indexing Words:
- Human heart
- Pressure-volume relations
- Cardiomyopathies
- Elastic stiffness
- Hypertrophy
- Compliance
- Coronary artery disease

The quantitation of left ventricular wall stiffness continues to be a problem of concern to the cardiologist. Unfortunately, progress toward a solution to this problem has been impeded by several factors: 1) an imprecise definition of elastic stiffness as employed in the current literature and 2) the technical difficulty associated with the simultaneous measurement of left ventricular pressure and volume throughout diastole.

Unlike the term "contractility," which has assumed a multitude of meanings over the years, the quantity elastic stiffness has a precise meaning when defined in terms of stress and strain (as employed in the theory of elasticity). If we define stress to be the force per unit cross-sectional area of a material and strain to be a change in length per unit length (generally the fractional change from the original or unstressed dimensions), then elastic stiffness is represented by the instantaneous change in stress with respect to an instantaneous change in strain. More specifically, if the material is biological, the stress-strain relation (fig. 1) is curvilinear and the elastic stiffness (which is a function of the stress) is given by the slope of the tangent at any point of this stress-strain curve. With the rapid progress being made in instrument technology it is now possible to collect simultaneous pressure-volume data. Thus in principle, earlier problems associated with the quantitation of elastic stiffness appear to be circumvented.

In earlier studies1,2 it was shown that quantities such as dP/dV, (dP/dV)/P and end-diastolic pressure divided by end-diastolic volume (EDP/EDV) individually represent only one of several determinants of elastic stiffness of the left ventricle. Other important determinants are wall stress and ventricular volume-wall mass ratio. Although the quantity dP/dV (termed elastance) is some measure of the resistance of a hollow elastic structure to deformation, it does not

From the Division of Mathematical Biology and Cardiovascular Division, Department of Medicine, Harvard Medical School and Peter Bent Brigham Hospital, Boston, Massachusetts.

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Dr. Levine is a Fellow of the Massachusetts Heart Association.

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Address for reprints: Dr. Israel Mirsky, Peter Bent Brigham Hospital, 721 Huntington Avenue, Boston, Massachusetts 02115.

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possess appropriate normalization properties which allow for valid patient-to-patient comparison. In an attempt at normalization, some investigators have divided this quantity by P. The resulting expression (dP/dV)/P which is essentially the slope of the dP/dV versus P relation, displays considerable variability within a group of patients, and therefore may not be a sensitive index. This is evident from previous studies which indicate that values of (dP/dV)/P for patients with congestive cardiomyopathy are lower than those for patients with normal ventricles. As for the expression EDP/EDV, it represents the slope of a line joining the origin (P = 0, V = 0) to the end-diastolic point and does not have any specific physiological meaning since the P-V curve does not pass through the origin.

The present study, in contrast to the previous study, is concerned with the development of a stiffness formula based on the more appropriate ellipsoidal geometry and its application to patients with coronary artery disease (CAD) and primary myocardial disease. Such studies will allow for the development of a standard index of stiffness which may then be used as a base to obtain correlates based on non-invasive techniques. Furthermore, these concepts can be extended to the more important problem of the assessment of segmental wall stiffness encountered in many patients with coronary heart disease.

Methods and Materials

Twenty-two patients with primary myocardial disease and 34 patients with coronary artery disease were studied. Ten patients found to be normal at diagnostic catheterization served as controls. The criteria for the diagnosis of primary myocardial disease are the same as those previously employed by Kreulen, Gorlin, and Herman. These patients were classified as Type I (normal contraction patterns and elevated end-diastolic pressure), Type II (hypertrophy without obstruction), and Type III (hypokinetic and/or asynergic).

With patients in a 30° right anterior oblique (RAO) projection, 40-50 ml of 76% meglumine sodium diatrizoate were injected over 3-4 sec into the left ventricle using multiholed catheters and a Viamonte Power injector. The ventriculograms were recorded on 16 mm cine film at a speed of 60-100 frames/sec. Ventricular silhouettes were calibrated to life-size using a filmed grid of known dimensions. Chamber volumes were calculated by the area-length method and left ventricular end-diastolic pressure was measured at high gain using a fluid-filled catheter system and Statham P23db transducer. Wall thickness was measured on the anterior surface of the left ventricle one third of the distance from the apex, and left ventricular mass was calculated according to a modification of the method of Ruckley et al. Left ventricular hypertrophy was diagnosed from the ECG employing the criteria of Sokolow and Lyon.

Since it was not possible to obtain simultaneous pressure-volume data throughout diastole in the present study, the pressure-volume relationship suggested by Gaasch et al. was adopted. These investigations employed the empirically derived expression P = 0.43e^{0.9} which was obtained on the basis of data taken from intact dog hearts. The limitations of this simplified expression will be discussed later.

Theoretical Considerations.

1. Expressions for elastic stiffness

In this section we present expressions for the passive elastic stiffness and stiffness constant k based on the ellipsoidal and spherical geometry assumption for the left ventricle. However, only the analysis for the ellipsoidal geometry will be discussed here in detail.

Ellipsoidal geometry

We seek here an expression for elastic stiffness in the form \( \frac{\partial \sigma}{\partial \varepsilon} = k_0 \sigma_0 + c_0 \) based on the assumptions: 1) the ventricular wall material is incompressible, i.e., the wall volume remains constant throughout the cardiac cycle; 2) instantaneous stresses \( \sigma_0 \) and strains \( \varepsilon \) at the midwall of the equator of an ellipsoid of revolution represent the over-all mechanical behavior of the ventricle since the fibers are predominantly circumferential in this midwall region.

In particular, the circumferential midwall stress \( \sigma_0 \) may be approximated by

\[
\sigma_0 = P(B/h)(1 - h/2B - B^2/2A^2)
\]

where A, B are respectively the semi-major and semi-minor axes at the midwall, h is the wall thickness, and P is the left ventricular pressure. For the evaluation of the elastic stiffness \( E_0 \), we employ increments of the instantaneous stress \( \sigma_0 \) and strain \( \varepsilon \) where \( \delta \sigma_0 \) is obtained by differentiation of the above expression and \( \delta \varepsilon = dB/B \).

It is shown in Appendix 1 that the elastic stiffness \( E_0 \) may be approximated by the expression

- **Figure 1**

Stress-strain and elastic stiffness versus stress relationships. Materials which obey Hooke’s law are characterized by a linear relationship and the slope E of this relationship is designated as Young’s modulus, E. For biological materials, the stress-strain relationship is curvilinear and usually of an exponential form. The slope at any point of this curve denotes the elastic stiffness for each stress level. \( \sigma \) is stress and \( \varepsilon \) is strain. The elastic stiffness versus stress relation is shown in the right panel. If the stress-strain relationship is exponential, the stiffness-stress relation is linear and the slope (k) of this line is termed the elastic stiffness constant. The intercept of this line on the vertical axis is denoted by c and represents approximately the stiffness of the muscle in the resting state. Note that if the material obeys Hooke’s law, the elastic stiffness is constant and independent of the stress level.
### Table 1

**Hemodynamic Data and Elastic Stiffnesses* for Specific Groups of Patients**

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*Includes 10 patients.

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ASSESSMENT OF LV STIFFNESS

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<td>19.6</td>
<td>1712</td>
<td>2570</td>
<td>0.7</td>
</tr>
<tr>
<td>34</td>
<td>16</td>
<td>160</td>
<td>1.20</td>
<td>0.76</td>
<td>42.6</td>
<td>65.0</td>
<td>18.1</td>
<td>17.9</td>
<td>772</td>
<td>1166</td>
<td>0.7</td>
</tr>
<tr>
<td>Mean</td>
<td>12.1</td>
<td>93</td>
<td>0.86</td>
<td>0.65</td>
<td>24.6</td>
<td>37.4</td>
<td>18.1</td>
<td>17.8</td>
<td>456</td>
<td>684</td>
<td>1.0</td>
</tr>
</tbody>
</table>

\(E_w = k_e \sigma_e + c_e\)

\(= \delta (\alpha_e/P) V (dP/dV) + (1 + \gamma) \sigma_e/(1 - h/2Bc)\)

Where

\(k_e = \delta (\alpha_{Vw} + (1 + \gamma)/(1 - h/2Bc))\)

\(c_e = \beta\delta V_{w} (\sigma_e/P)_{ed}\)

\(dP/dV = \alpha P + \beta\)

And the constants \(c, \delta, \gamma\) etc. are defined in Appendix 1. The subscripts e and ed, respectively denote ellipsoid and end diastole. Although the above expressions for \(c_e\) and \(k_e\) are approximate, they are useful when only end-diastolic quantities are available. If P-V data is available throughout diastole, more accurate expressions can be obtained by plotting \(E_e\) versus \(\sigma_e\) and employing a linear regression analysis. As noted previously,1 and from Appendix 1, the elastic stiffness of the left ventricle depends on several important physiological parameters: i) \(dP/dV\), ii) wall stress \(\sigma_e\), and iii) ventricular volume-mass ratio \(V_in V_w\) through the constants \(\gamma\) and \(\delta\). The volume-mass ratio \(V_in V_w\), considered recently by Field et al.,13 is a very important parameter and has been omitted by most investigators in their analysis of elastic stiffness.

Spherical geometry

For completeness of the present discussion, we repeat here the corresponding results for the spherical ventricle obtained from earlier studies.1 The elastic stiffness is again expressed in the form \(E_w = k_e \sigma_e + c_e\)

Where

\(k_{se} = 3[1 + \alpha V_{w}] [1 + (V_{w}/V)] a^2/(a^2 + b^2)_{ed}\)

\(c_e = 3\delta V_{w} (\sigma_e/P)_{ed} [1 + (V_{w}/V)] a^2/(a^2 + b^2)_{ed}\)

\(\sigma_e = P(V/V_w)/(1 + b^2/2R^2)\)

And a, b, and R are respectively the internal, external, and midwall radii of the sphere.

Results

In table 1 the hemodynamic data and elastic stiffness \(E_w, E_s\) and stiffness constants \(k_e, k_s\) are presented for each patient. Mean values of \(k_{se}\) and \(E_{se}\) for the various groups are 14.8 ± 0.7 and 329 ± 54 g/cm² for the normal group; 18.2 ± 0.5 and 1133 ± 127 g/cm² for Type I; 22.4 ± 1.2 and 833 ± 150 g/cm² for Type II; 18.7 ± 0.6 and 1623 ± 348 g/m² for Type III; and 17.8 ± 0.3 and 684 ± 80 g/cm³ for the group with coronary artery disease. Mean values for wall thickness \(h_{m}\) are 0.90 ± 0.05 cm for the normal group, 0.85 ± 0.06 cm for Type I, 1.53 ± 0.15 cm for Type II, 1.06 ± 0.14 cm.
End-diastolic elastic stiffness $E_e$ (based on ellipsoidal geometry) for ten controls (normal), 34 patients with coronary artery disease (CAD), and 22 patients with cardiomyopathy who were classified as Type I (normal contraction pattern and elevated end-diastolic pressure), Type II (hypertrophic without obstruction) or Type III (hypokineti and/or asynergic). There is a good separation between the normals and Type III (all of whom had ejection fractions less than 50%). The one patient in Type III with a normal stiffness level is alive and well seven years from study. Although the mean value of $E_e$ for the CAD group is higher than that for the normal group, all CAD patients with end-diastolic pressures $\geq$ 12 mm Hg had normal stiffness levels. Type I patients had elevated stiffness levels even though contraction patterns and ejection fractions were normal. Several patients in Type II (hypertrophy) displayed normal stiffnesses in the presence of elevated end-diastolic pressures.

Discussion of Results

The manifold results of these studies are based on several simplifying assumptions, one being that pressure-volume and stress-strain relations for the intact human heart are of an exponential form. This has been shown by several investigators to be valid for the intact dog heart, but as yet, its validity in man has not been established.

Relative to the stiffness constants $k_e$, $k$, the results indicate that 1) they are unaffected by the assumed geometry of the ventricle and the stress formula employed; 2) stiffness constants are primarily dependent on the P-V relations, diastolic pressure, and the ventricular volume-mass ratio V/Vw. In particular, k is elevated in patients with hypertrophy due to the increased wall mass and elevated dP/dV; and 3) values of k for the cardiomyopathy groups Type I and Type III are higher than those for the normal group due to the elevated end-diastolic pressures. Thus, elevated k values combined with abnormal end-diastolic stresses may be associated with fibrosed ventricles. A similar finding is likely to be observed with scarred ventricles cm for Type III, and 1.01 $\pm$ 0.03 cm for patients with coronary artery disease.

In figures 2-6, the various indices of stiffness evaluated at end diastole are graphically displayed. Figure 7 demonstrates the good correlation ($r = 0.99$) obtained between $E_e$ (stiffness based on ellipsoid) and $E_s$ (stiffness based on a sphere). The poor correlation ($r = -0.52$) observed in figure 8 between elastic stiffness $E_s$ and the ejection fraction (EF) confirms similar findings obtained with contractility indices and EF.

The index $dP/dV$ at end diastole is displayed for patients with normal ventricles, coronary artery disease (CAD), and primary myocardial disease (Types I, II and III). Note the poor separation between the normal and Type III groups.
ASSESSMENT OF LV STIFFNESS

The index (dP/dV)/P at end diastole is shown for the five groups of patients studied. There is no significant difference between the mean values of normal, CAD, Type I, and Type II groups. Furthermore, the mean value is lowest for the Type III group, an unexpected result.

1

Figure 5

The index (dP/dV)/P at end diastole is shown for the five groups of patients studied. There is no significant difference between the mean values of normal, CAD, Type I, and Type II groups. Furthermore, the mean value is lowest for the Type III group, an unexpected result.

present in many patients with coronary artery disease. These conclusions are supported in part by Parmley et al. 17 who quantitated the length-tension relations of resected human ventricular aneurysms and showed that the k values for fibrous aneurysms were markedly greater than muscular aneurysms.

of particular importance in the present analysis is the elastic stiffness E which is approximated by the expression E = kσ (σ = midwall stress). As observed in figure 2 and table 1, several points are worthy of note: 1) there is a good separation between the normal

group and Type III (all of whom had ejection fractions less than 50%); 2) although Type I patients had normal ejection fractions and normal contraction patterns, their stiffnesses were elevated; 3) normal stiffness levels were observed in coronary artery disease patients with end-diastolic pressures ≤ 12 mm Hg. In fact, there was a good correlation between E and end-diastolic pressure (r = 0.87) except for several patients with hypertension; 4) it is evident from the wall thicknesses of Type II patients that hypertrophy is present. The finding that normal stiffness levels may be present in patients with hypertrophy is more dramatically shown in figure 3.

Results displayed in figure 3 are based on data from earlier studies 3 and include 13 normals, seven patients with idiopathic hypertrophic subaortic stenosis (IHSS), and six patients with congestive cardiomyopathy (CC). Again there is a good separation between the normals and the cardiomyopathies; however, of particular interest is the result that all patients with IHSS were operating at normal stiffness levels even in the presence of elevated end-diastolic pressures. This finding was not predicted by the index V(dP/dV) shown on the right panel of figure 3 or by any of the other "indices of stiffness," and the discrepancy may well be due to the simple P-V relation employed. The finding of normal stiffness in these patients with IHSS may not be surprising since the stiffness constant k increases with the mass-volume ratio Vm/V but the wall stress σ decreases faster, the normal stiffness levels (E = kσ).

In figures 4-6, indices currently being employed are displayed on the basis of the present data. It is quite apparent that dP/dV, (dP/dV)/P and to a lesser ex-

Figure 6

Volume elasticity V(dP/dV) at end diastole is shown for the five groups of patients. On a group basis, the results are similar qualitatively to those shown in figure 2 for E (elastic stiffness based on ellipsoidal geometry); however, there are several discrepancies on an individual basis. As stated in the text, this index is a function of pressure only and therefore may be unreliable in patients with hypertrophy (see fig. 3).

Figure 7

The elastic stiffness E (based on ellipsoidal geometry) versus the stiffness E (based on spherical geometry). Although the assumption of a sphere always underestimates the wall stiffness, the qualitative results are unaltered as evidenced by the good correlation between E and E (r = 0.99).

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tent V(dP/dV) (all evaluated at end diastole) display considerable overlap between the various groups. These indices have some clinical value (albeit limited) in the sense that they may indicate directional changes in elastic stiffness but do not accurately quantitate stiffness.

The good correlation \( r = 0.99 \) obtained between stiffnesses based on the spherical geometry and ellipsoidal geometry (fig. 7) indicates that qualitative results are unaltered by the LV geometry assumption. As observed in figure 8, 1) poor ejection fractions are generally associated with elevated stiffnesses; 2) patients with normal stiffnesses have normal ejection fractions. But there are many patients with normal ejection fractions who have elevated stiffness values that suggest that the ejection fraction may not be a reliable guide to LV stiffness.

Limitations of the Analysis

There are several limitations to the present analysis and these are outlined as follows:

a) The assumption that circumferential midwall stress for the ellipsoidal geometry is representative of the average fiber stress may be valid at the equator only. Streeter et al.\(^\text{18}\) have shown that at end diastole, the circumferential stress predominates in the middle 90% of the wall and meridional (or longitudinal) stresses predominate in the outer 20%. Furthermore, McHale and Greenfield\(^\text{19}\) indicate that ellipsoidal stress formulae agree reasonably well with measured values of average wall stresses. On the other hand, Janz and Grimm\(^\text{20}\) employing a rather sophisticated mathematical model based on observations of Streeter et al.,\(^\text{18}\) show that pressure-fiber length curves vary for the inner, outer, and midwall fibers and also from base to apex. Thus the present model at best can lead to only approximate global values of wall stiffness and does not quantitate stiffness of isolated segments.

b) Determinations of ventricular wall mass are subject to error especially in patients with hypertrophy. In particular, patients with IHSS have been shown to reveal an asymmetry of hypertrophy.\(^\text{21}\) It may therefore be advisable to employ both angiography and echocardiography for this group of patients.

c) The assumption for the P-V relation in the form \( P = 0.43e^{0.9} \) cannot be justified for each patient since the numerical factor was obtained from normal dog data. As a consequence, \( V \, dP/dV \) is a function of pressure only. In their recent studies, Gaasch et al.\(^\text{22}\) indicated considerable variability in the numerical factor (range 0.14 to 1.49) even though the mean values of stiffness for the various groups were not statistically significant from those based on the factor 0.43. Substantial errors in the measurement of both wall mass and P-V relations can lead to significant errors in the values for wall stiffness. An error analysis presented in Appendix 2 indicates that a 100% overestimation of wall mass and a 300% error in the numerical factor (0.1 in place of 0.43), may result in a 50% underestimation of wall stiffness. If, in addition, the geometry of left ventricles in patients with IHSS is closer to being cylindrical rather than ellipsoidal (thus increasing wall stress), it is possible that the stiffness values in some of these patients could be abnormal. With the advent of instrumentation such as the Millar catheter, it is now possible to obtain more reliable P-V data during diastole so that the above simplification is no longer necessary.

d) In patients whose clinical findings suggest asynergy, biplane ventriculography allows a more comprehensive evaluation of left ventricular wall motion. However, the studies by Cohn et al.\(^\text{23}\) indicate that ejection fractions determined by single plane and biplane ventriculography agreed reasonably well in patients with and without asynergy. Since the present analysis assumes a uniform expansion of the ventricle to take place, valid objections may be raised in applying this method to patients with asynergy of contraction or segmental disease.
segmental pressure-volume curves. This approach, currently being carried out in our laboratory, will allow for the quantitation of wall stiffness in isolated areas of akinesis and dyskinesis.

Conclusions

Simple indices such as dP/dV, (dP/dV)/P, EDP/EDV, etc. may detect directional changes in stiffness but do not themselves quantitate elastic stiffness. More appropriately, evaluation of elastic stiffness should be based on the concepts of stress and strain as defined by the theory of elasticity which has been developed over the past 300 years. Because of their normalization properties, terms such as stress and strain allow for more meaningful comparisons of elastic stiffness to be made between ventricles of different size and shape.

The stiffness constant k is representative of the parallel elastic element in the 3-element model of cardiac muscle. It would therefore appear that the parallel elastic stiffness constant increases in hypertrophy. This result is in agreement with the findings of Bing et al., Spann et al., and Hamrell and Alpert in their studies of experimental hypertrophy in isolated heart muscle. The fact that some patients with hypertrophied ventricles may have normal stiffness values does not imply that hypertrophied ventricles are compliant. On the contrary, it means that these patients are operating at low stress levels of steep stiffness-stress curves (fig. 9).

Although the studies by Kreulen et al. show that the cardiomyopathies in general may be classified on the basis of the left ventriculogram, there may not always be a correlation between elastic stiffness and contraction patterns as evidenced by the results of Type I patients. Furthermore, normal stiffness levels may be associated even in patients with coronary artery disease. Finally, invalid interpretation and quantitation of elastic stiffness of the left ventricle may lead to misdiagnosis of heart failure if current “indices of stiffness” are employed.

Appendix 1

Evaluation of elastic stiffness and stiffness constant for an ellipsoid of revolution

It can be shown from the studies of Mirsky that the midwall circumferential stress \( \sigma_w \) at the equator of an ellipsoid of revolution may be approximated by the expression

\[
\sigma_w = P(B/h)(c_1 - h/2B)
\]

where \( h \) is the wall thickness, \( A, B \) are respectively the midwall semi-major and semi-minor axes, and \( c_1 = 1 - B^2/2A^2 \) which is approximately constant near end diastole.

The instantaneous increment in stress is therefore

\[
d\sigma_w = (\sigma_w/P)dP + (c_1/P)dB - (c_1B/h^2)dh
\]

The left ventricular volume \( V = (4\pi/3)(A - h/2)(B - h/2)^2 \), hence the increment of volume is given by

\[
dV = (2d(B - h/2) + dA - h/2) \frac{h}{(A - h/2)}
\]

If we assume that the ratio \( B/A \) is approximately constant near end diastole, i.e., \( dA/dB = B/A \), a detailed analysis yields the following relations:

\[
dh/h = -\gamma (dB/B),
\]

where

\[
\gamma = \frac{(\alpha_sV_w - \beta_sV)/(\alpha_sV_w + \beta_sV)}{(1 + \gamma h/2A)(1 + h/2A)}
\]

\[
\delta = (2 + \gamma h/B)(1 + h/2B + h^2/4B^2 + (1 + \gamma h/2A)(1 + h/2A)
\]

\[
\beta_s = \frac{(h/2)(2 + h^2/2B^2) + h/A}{(h/B)(1 - h/2B + h^2/4B^2) + (1/2A)(1 - h/2A)}
\]

Thus

\[
d\sigma_w = (\sigma_w/P)dP + \sigma_w(1 + \gamma)(dB/B)/(1 - h/2B_c)
\]

The instantaneous increment in the midwall strain is \( d\varepsilon = d/P \), therefore the elastic stiffness may be written in the form

\[
\varepsilon_s = \frac{d\sigma_w}{d\varepsilon} = \frac{\varepsilon_s}{\varepsilon_s} = \frac{\varepsilon_s}{\varepsilon_s}
\]

\[
= \frac{\varepsilon_s}{\varepsilon_s} = \frac{\varepsilon_s}{\varepsilon_s} = \frac{\varepsilon_s}{\varepsilon_s}
\]

\[
= \frac{\varepsilon_s}{\varepsilon_s} = \frac{\varepsilon_s}{\varepsilon_s} = \frac{\varepsilon_s}{\varepsilon_s}
\]

If the P-V relation is expressed as \( dP/dV = \alpha P + \beta \), the constants \( k_w, c_w \) may be approximated by the expressions

\[
k_w = \frac{\delta(\alpha_sV_w)}{(1 + \gamma)/(1 - h/2B_c)}
\]

\[
c_w = \frac{\delta V_w}{\alpha_s/P} = \frac{\delta V_w}{\alpha_s/P}
\]

which is the desired result.

Appendix 2

Error analysis for elastic stiffness based on a spherical geometry.

Figure 9

Stiffness-stress relationships for a typical normal patient (N), a patient with inappropriate hypertrophy (IH), and a patient with congestive cardiomyopathy (CC). E_n, E_m, and Ecc refer to the stiffnesses for the respective groups. The stiffness constants k are respectively 18.9, 28.1, and 23.0 and are obtained by a linear regression analysis assuming the pressure-volume relations to be valid over pressure ranges 5 to 15 mm Hg, 10 to 25 mm Hg and 15 to 40 mm Hg respectively. The numbers in parentheses refer to the stiffness constants (based on sphere) obtained from table 3 of reference 1. At a common stress level, the elastic stiffness (E ~ k) is highest for the IH group since k is highest for the hyper trophy group. However, diastolic stresses in this group are normal to subnormal, hence the operating stiffness levels are close to normal. (Reproduced from Circulation Research 33: 233, 1973, with permission of the American Heart Association Inc.)

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Appendix 2

Error analysis for elastic stiffness based on a spherical geometry.

The expression for the elastic stiffness based on a spherical geometry may be written in the form

\[ E_t = P \left( \frac{dF(v)}{dv} \right) \left[ 1 + \left( \frac{V}{P} \right) \frac{dP}{dV} \right] \]

where \( v = V/V_0 \) is the volume-mass ratio and \( F(v) = 3V \left[ 1 + v + \frac{a}{v(v + 1)} \right] \times \left[ 1 + v + \frac{a}{v(v + 1)^{1/2}} \right] \). Values of the quantity \( F(v) \) for various values of the volume-mass ratio \( v \) are given below:

\[
\begin{array}{c|c}
\nu & F(v) \\
\hline
0.25 & 3.04 \\
0.5 & 4.5 \\
1.0 & 7.1 \\
1.5 & 9.4 \\
2.0 & 11.7 \\
\end{array}
\]

As a numerical example, consider first that errors occur in the measurement of wall mass only and that the true value of \( v = 0.5 \). If the wall mass is overestimated by 100%, the observed value of \( v = 0.25 \) and the relative error incurred in the elastic stiffness is \((4.5 - 3.04)/4.5\) or a reduction in stiffness of 32.5%.

Consider now that errors occur only in the measurement of the P-V relations which are given by \( P = b \nu^n \). Hence we are interested in the error involved in the quantity

\[ 1 + \left( \frac{V}{P} \right) \frac{dP}{dV} = 1 + \frac{a}{V} + \frac{a}{V} \left( \frac{1}{V} \right) \]

If the true value of \( b = 0.1 \) instead of 0.45 and the end-diastolic pressure \( P = 20 \text{ mm Hg} \), the relative error is \((6.3 - 4.84)/6.3\) or a reduction in stiffness of 23%. Note that if the true value of \( b = 2 \) there would be an increase in stiffness by 47%.

Assuming now that errors occur both in the measurement of mass and the constant \( b \) so as to maximize the error, the observed value of stiffness \( 20 \times 3.04 \times 4.84 = 295 \text{ mm Hg} \) compared to the true value of \( 20 \times 4.5 \times 6.3 = 567 \text{ mm Hg} \). Hence the relative error = \((567 - 295)/567\) or a reduction in stiffness of 48%.

The above example is cited as a possibility that may arise in the quantitation of wall stiffness in patients with IHSS.

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Assessment of Left Ventricular Stiffness in Primary Myocardial Disease and Coronary Artery Disease
ISRAEL MIRSKY, PETER F. COHN, JAY A. LEVINE, RICHARD GORLIN, MICHAEL V. HERMAN, THOMAS H. KREULEN and EDMUND H. SONNENBLICK

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EDITORIAL


Correction

Mirsky I et al.: Circulation 50: 128, 1974. On page 135, the first sentence should read, “For such patients the method must be modified by performing biplane studies and quantitating segmental pressure-volume curves.”

Also on page 135, in Appendix 1, the constants $\alpha_1$, $\beta_1$, and $\alpha_2$ are:

$\alpha_1 = 3 - h/B - h/2A + h^2/2B^2$

$\beta_1 = h/A + 2h/B$

$\alpha_2 = (h/B) (1 - h/2B + h^2/4B^2) + (h/2A) (1 - h/2A)$