Spatial Parameters and Shape Factors of the Normal Atrial Vectocardiogram and Its Scalar Components

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SUMMARY

New quantitative norms for clinical evaluation of the atrial vectorcardiogram are presented. Real-time computer processing digitizes atrial electrocardiograms, reduces random noise content, rectifies base-line configuration, and corrects preamplifier distortion. Axial-system leads of 106 normal persons were so treated. The basic information derived includes spatial distribution and magnitudes of P, Tp, and polar P vectors; length, width, and planarity of P loop; spatial P-Tp angles; and the Eulerian angles of P-loop normalization. Normal P vectors show predominant leftward, inferior, and anterior orientation, with opposing Tp direction. Polar P vectors show predominant superior, anterior, and leftward clustering. The scatter of parameters in the orthogonal-lead frontal plane is about the same as for the tetrahedral system. Similarly, the atrial deflections of normals show equally rich notching in both systems. Resolution of the P-wave population into uncorrelated components, followed by resynthesis from these principal factor wave forms, revealed a fairly continuous “spectrum” of signal configurations. This technique, and extensive attempts by alternate means, failed to support the view that normal P waves can be separated into distinctive right and left atrial components.

Additional Indexing Words:

P-wave analysis
P-wave normalization
Atrial repolarization
Signal averaging
Electrocardiographic processing
Computer applications

In the past it has been difficult for clinical investigators to study the atrial vectorcardiogram and its scalar components in fine-grained detail. The application of modern signal processing methods has recently opened up this relatively remote area of cardiac electrophysiology by overcoming many of the technical difficulties which had formerly obstructed access. During the past year we have applied such methods to the atrial vector signals of 106 normal subjects. The results of our observations are recorded in this report with the hope that they will later serve as standards of normality against which data obtained from individuals with heart disease can be compared.

Methods

The corrected orthogonal leads of 115 predominantly youthful subjects were recorded on magnetic tape, using the axial lead system of McFee and Parungao. A history of freedom from heart disease was obtained from each subject, and this information was supplemented by a clinical examination which included determination of blood pressure, cardiac auscultation, and evaluation by palpation and percussion. Each set of orthogonal leads, together with the bipolar extremity lead in which QRS amplitude
was greatest for that particular subject, was recorded continuously over a 5-min period. Recording was done on a mobile equipment cart which included among its contents a Hewlett-Packard model 3900 tape unit, four Grass P511 physiological amplifiers, a patient cable that terminated in a bank of four Grass HIP511 high-impedance input probes, and a DC-energized mercury-in-rubber type of strain-gauge pneumograph. Recording was done with the subject lying comfortably in an adjustable hospital bed, with due attention paid to adequate skin preparation and careful electrode application. The technical quality of the recorded signals was visually monitored throughout the procedure by observing them on a cathode-ray oscilloscope built into the recording cart. A calibration signal was recorded from a carefully standardized free-running multivibrator which produced a 1-mv peak-to-peak square wave at a frequency of 10 Hz.

The tape-recorded signals were processed on a laboratory-oriented digital computer* by methods which, for the most part, have been previously described. On playback the X, Y, and Z wave forms were digitized at the rate of 500 samples per second and stored in buffer memory. The fourth recorded channel, usually lead II, was digitized at an identical rate after having first been time-differentiated and heavily smoothed by analog circuits. The amplitude of this modified signal was constantly tested by digital "compare" logic, and the points at which each QRS complex first exceeded a preset value were taken as the temporal fiducial marks of successive cardiac cycles.

A major objective of the signal processing was to improve wave-form quality by averaging waves over 50 cardiac cycles. This was accomplished by a two-pass procedure which was similar to one employed in a previous study. On the first pass the electrocardiogram was checked for evidence of metastable or unstable or both types of electrical behavior by a real-time application of wave-form correlation logic to successive P-T, complexes. After a dominant pattern of atrial wave form was established—as was possible in most cases—corresponding tolerance limits were entered into the computer, thus ensuring the rejection of all other patterns during the signal averaging (that is, second pass) portion of the program. Each 50-beat average was then subjected to a numerical equalization procedure which removed most of the low-frequency distortion which has been introduced by the capacitative interstage coupling networks of the preamplifiers.

Four of the original 115 records were excluded from further analysis because of abnormal auscultatory findings. Another five were excluded for various technical reasons: one because of excessive base-line wandering, one because of excessive P-wave variability, one because of R-R interval greater than the programmed maximum of 1.2 sec, and two because of variability in the orthogonal leads which was not adequately reflected in the monitor lead. Table 1 summarizes according to age grouping and sex the 106 subjects whose records were retained for complete study.

The beginning of the P wave and the offset of the P-R segment were visually determined from write-outs of the wave-forms on an incremental plotter. A third temporal mark, the dividing point between the end of the P and the beginning of T, was calculated as that sample at which the angle between the instantaneous vector and the mean atrial vector, computed cumulatively up to that point, changed from acute to oblique. The physiological basis for such a determination is challengeable since some portions of the atria will have begun to repolarize before depolarization has been completed in the atria as a whole. However, although this method of temporal demarcation is somewhat arbitrary, it avoids the confusion due to phase differences between the respective base-line crossings of P in individual scalar leads and thus provides a relatively simple way of determining a useful time parameter in a precisely repeatable manner.

### Results

A right-handed Cartesian coordinate system was used in which the X axis lay horizontally in the subject's frontal plane and extended positively toward his left; the Y axis lay vertically in the frontal plane, positive

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*PDP7, Digital Equipment Corporation, Maynard, Massachusetts.

### Table 1

**Distribution of Subjects by Age and Sex**

<table>
<thead>
<tr>
<th>Age (yr)</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>16</td>
<td>23</td>
<td>39</td>
</tr>
<tr>
<td>25-34</td>
<td>30</td>
<td>17</td>
<td>47</td>
</tr>
<tr>
<td>35-44</td>
<td>3</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>45-54</td>
<td>2</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>55-64</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>51</td>
<td>55</td>
<td>106</td>
</tr>
</tbody>
</table>
footward; and the Z axis was normal to the frontal plane, and directed positively toward the back. Extensive use was also made of a coordinate sphere as an aid in defining a variety of P-vectorcardiographic parameters. The polar axis of the sphere corresponds to the anatomic vertical axis and, therefore, to the electrical Y axis also. The equatorial plane has a horizontal lie, and contains the X and Z coordinate axes. Angles of elevation (that is, latitudes) are referred to the equator; positive values, to the footward side, and negative values are directed upward. Azimuth is reckoned from the left of the frontal plane, with the prime meridian passing through the positive half of the X axis (that is, leftward), and the 90° meridian through the negative Z axis (forward). Although these designations differ from the conventions ordinarily employed by engineers and mathematicians, they are in accord with usual vectorcardiographic notation.

The area under the P wave and P-R segment in each of the orthogonal leads was computed by the trapezoidal rule of numerical integration. For purposes of clinical analysis the P-R segments were treated as Tp deflections throughout the study, but with clear appreciation of the fact that atrial repolarization begins earlier than the onset of the P-R segment and continues well into the QRS complex. Transformation of the P and Tp vectors from Cartesian to spherical coordinates was performed in the usual manner by

\[
P = \sqrt{(P_x)^2 + (P_y)^2 + (P_z)^2}
\]

\[
\theta = -\tan^{-1} \left(\frac{P_z}{P_x}\right)
\]

\[
\phi = \tan^{-1} \left(\frac{P_y}{\sqrt{(P_x)^2 + (P_z)^2}}\right)
\]

where P is the magnitude of the spatial vector; Px, Py, and Pz are the areas under the deflections in leads X, Y, and Z, respectively; \(\theta\) is the angle of azimuth; and \(\phi\) is latitude. The same formulations apply to atrial repolarization forces, with scalar Tp-values substituted for P-wave components. A little care must be exercised in avoiding the quadrant ambiguities of angular measure which are implicit in equations 2 and 3.

A revealing overview of vector orientations is obtained by plotting radial projections of

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**Figure 1**

Transformation of coordinate sphere into a planar map, showing the spatial orientation of mean electrocardiographic P vectors (open circles) and Tp vectors (solid circles). The details of the spatial grouping which are evident in this map are described further in the text. (In the quadrant on the reader's lower right all circles except the rightmost are open, although dense clustering gives the impression that several may be solid.)

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their termini on the coordinate sphere referred to above. The value of the display is further enhanced by transformation of the coordinate sphere, in a manner similar to that employed by cartographers, to a planar map. Thus it can be quickly recognized from inspection of figure 1 that all but three P vectors (open circles) of this group of 106 normal subjects have a footward component of orientation, and that all but five T_p vectors (solid circles) have a headward component. It is further apparent that the P vectors as a group have fairly strong footward and leftward components of direction, with only mild predominance of forward orientation. All but eight of the T_p vectors have a backward component of orientation, and there is an approximately five-to-one predominance of rightward over leftward orientation.

The study included an analysis of the polar vectors associated with atrial depolarization. The polar vector is a spatial parameter which was proposed by Burger and Vaan9 and which, as ordinarily used, is applied to the QRS loop. The orientation of the vector may be defined as the axis of projection which produces maximal loop area, and this area, in turn, specifies the magnitude of the vector. Sense, or polarity, of this parameter is defined by the right-hand rule; that is, counterclockwise rotation of the vector loop generates positive polar-vector moment, and vice versa.

A fairly good approximation of the polar vector can be calculated from

\[
\begin{align*}
V_x &= \frac{1}{N} \sum_{i=1}^{N-1} (Y_{i+1}Z_i - Z_{i+1}Y_i) \\
V_y &= \frac{1}{N} \sum_{i=1}^{N-1} (Z_{i+1}X_i - X_{i+1}Z_i) \\
V_z &= \frac{1}{N} \sum_{i=1}^{N-1} (X_{i+1}Y_i - Y_{i+1}X_i),
\end{align*}
\]

where N is the number of digitized samples in the area of interest; \(V_x\), \(V_y\), and \(V_z\) are the X, Y, and Z components, respectively, of the polar vector; and the subscripted X-, Y-, and Z-symbols are the quantized values of the respective lead samples. Upon applying equation 4 to our P-wave data, we obtained a mean azimuth of 51.5° with a standard deviation of ±27.8°, and a mean latitude of −21.0° with a standard deviation of ±21.3°. The spatial orientations of the polar P vectors were fairly well clustered (fig. 2) as such.

**Figure 2**

Spherical coordinate map showing spatial orientation of polar-P vectors in a group of 106 normal subjects. Except for a few "outriders" the spatial directions are clustered to about the same extent as is seen with more customary parameters such as orientation of mean QRS and T vectors. Further description in text.
matters go in clinical vectorcardiography, with all but four cases contained within a spherical trapezoid bounded by the +20° and -70° circles of latitude, and the meridional circles corresponding to the 10° and 100° angles of azimuth.

A set of direction cosines was calculated for each atrial vector by

\[
\begin{align*}
\lambda &= \cos \alpha \sin \beta \\
\mu &= \cos \beta \\
\nu &= -\sin \alpha \sin \beta
\end{align*}
\]

where \(\alpha\) is azimuth, \(\beta\) is latitude, and \(\lambda, \mu,\) and \(\nu\) are the direction cosines. Each set of direction cosines represents the spatial orientation of a given vector, and the cumulative product of corresponding members of two sets gives the cosine of the angle between a pair of vectors. Spatial P-Tp angles of the 106 subjects were thus calculated, giving a mean value of 144.2° with a standard deviation of ±21.3°. Since virtually all of these angles are oblique, and yet cannot exceed 180°, their distribution is manifestly non-Gaussian (see fig. 8, lower panel). On the other hand this histogram can be fitted rather nicely by a Poisson distribution.

Spatial Distribution of Atrial Vectors

When magnitude as well as orientation is taken into consideration, each electrical vector can be represented by the location of its terminus in reference space. The entire set of vector termini therefore forms a three-dimensional scattergram. It is convenient to simplify such treatment by projecting the spatial array of points on each of the coordinate planes, and then dealing individually with the planar projections as two-dimensional scattergrams. Treatment is also simplified by assuming that
the spatial distribution is Gaussian, in which case the centroid of the array (that is, the point whose respective coordinates are simple averages of the entire group) is a statistically valid representation of the mean vector.

As is illustrated in figures 3 through 5, we handled the P, Tp, and polar-P vectors of our subjects in this way. Each panel of each of these illustrations shows two concentric ellipses centered about the individual mean vector termini. The length of a line drawn to the inner and outer ellipses from their center represents 1 and 2 standard deviations of distribution in the direction of that particular line. The major and minor axes of the ellipses represent, respectively, the directions of greatest and least scatter and are known therefore as the principal axes of distribution. In principle, the inner ellipse in each panel en-

![Spatial distribution of the polar P vectors of 106 normal subjects. The spatial magnitude of the vector signifies the area of the P loop when viewed in its most open aspect. Individual components of a spatial polar P vector are the area of the loop when viewed along the corresponding axis. Therefore the X component, for example, is the area of the loop in its YZ-plane projection. The algebraic sign of the area is determined by application of the right-hand rule. The scale marking of 10,000 "square microvolts" corresponds to the area of 0.1 inch square for a vectorcardiogram recorded at the commonly employed sensitivity of 1 inch per millivolt. Further description in the text and table 2.](image)

compasses 39.3% of all points, and the outer ellipse, 86.5%. An ellipse with semi-axis lengths of 2.45 standard deviations would account for 95% of the observations. The quantitative information derived from this analysis is contained in table 2.

**Axis Normalization**

Normalization of axes may be defined as the rotation of vector forces to some standardized orientation within the Cartesian frame of references. We performed this procedure on our P-vector data, using three different sets of criteria to define axis normalization. In each case the results were determined as the successive angular values of a "three knob" rotation.

In method I the first angle of rotation, A, is that which revolves the P vector about the Y axis until it lies in the XY plane. The second rotation, B, then rotates the P vector about the Z axis until it is colinear with the X axis. The third rotation, C, is performed about the X axis until a condition of minimum energy is achieved in the final scalar Y lead. Ambiguity in specifying the quadrant of angle C is removed by the further condition that the final scalar Z lead be "minus-plus" diphasic. In relation to the original spatial orientation of the P vector, angle A is the negative of azimuth, and angle B is the same as latitude. Clockwise rotation about the X axis defines a positive value of angle C.

The execution of method II is based on the principle of eigenvectors. In brief, an auxiliary set of mutually orthogonal axes, U, V, and W, is determined in which a maximal amount of signal energy is present in scalar lead U, a minimum amount, in scalar V, and intermediate content, in lead W. These manipulations are followed by three rotations which are similar to those of method I: (A) about Y until the U axis lies in the XY plane, (B) about Z until U becomes colinear with X, and (C) about X until V lines up with the Y axis, and the same quadrant rule is observed as in the first method.

In method III the first two rotations are performed in exactly the same manner as in method I. The third angle of rotation, about
NORMAL ATRIAL VECTORCARDIOGRAM

Table 2

<table>
<thead>
<tr>
<th>Average components of group</th>
<th>Plane</th>
<th>Planar magnitude</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( \bar{z} )</th>
<th>Ecc.</th>
<th>Ang.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{x} = 2.55 \mu \text{v-sec} )</td>
<td>Frontal</td>
<td>5.55</td>
<td>2.72</td>
<td>1.31</td>
<td>1.88</td>
<td>0.877</td>
<td>74.6°</td>
</tr>
<tr>
<td>( \bar{y} = 4.93 )</td>
<td>Lt. sag.</td>
<td>4.95</td>
<td>2.65</td>
<td>1.06</td>
<td>1.68</td>
<td>0.916</td>
<td>88.5</td>
</tr>
<tr>
<td>( \bar{z} = -0.50 )</td>
<td>Horizon.</td>
<td>2.60</td>
<td>1.46</td>
<td>1.06</td>
<td>1.24</td>
<td>0.688</td>
<td>-6.6</td>
</tr>
<tr>
<td>( \bar{x} = -0.75 \mu \text{v-sec} )</td>
<td>Frontal</td>
<td>1.62</td>
<td>1.23</td>
<td>0.91</td>
<td>1.06</td>
<td>0.675</td>
<td>-46.5°</td>
</tr>
<tr>
<td>( \bar{y} = -1.44 )</td>
<td>Lt. sag.</td>
<td>1.64</td>
<td>1.18</td>
<td>0.69</td>
<td>0.90</td>
<td>0.811</td>
<td>62.7</td>
</tr>
<tr>
<td>( \bar{z} = 0.79 )</td>
<td>Horizon.</td>
<td>1.09</td>
<td>1.17</td>
<td>0.67</td>
<td>0.89</td>
<td>0.820</td>
<td>-28.9</td>
</tr>
<tr>
<td>( \bar{x} = 3542 (\mu \text{v})^2 )</td>
<td>Frontal</td>
<td>4035</td>
<td>3013</td>
<td>2264</td>
<td>2612</td>
<td>0.660</td>
<td>-5.6°</td>
</tr>
<tr>
<td>( \bar{y} = -1933 )</td>
<td>Lt. sag.</td>
<td>4925</td>
<td>2830</td>
<td>2205</td>
<td>2498</td>
<td>0.627</td>
<td>18.0</td>
</tr>
<tr>
<td>( \bar{z} = -4530 )</td>
<td>Horizon.</td>
<td>5751</td>
<td>3555</td>
<td>2028</td>
<td>2685</td>
<td>0.821</td>
<td>-40.5°</td>
</tr>
</tbody>
</table>

Symbols and abbreviations: \( \mu \text{v-sec} \) = microvolt seconds; \( (\mu \text{v})^2 \) = "square microvolts"; \( \sigma_1 \) = standard deviation along major principal axis of distribution; \( \sigma_2 \) = standard deviation along minor principal axis of distribution; \( \bar{z} \) = geometric mean of \( \sigma_1 \) and \( \sigma_2 \); ecc. = eccentricity of ellipses of distribution; ang. = angle of inclination of major axes as referred to the horizontal axis of each panel in figures 3 through 5.

The X axis, is defined by the end point condition

\[
\int_{t} y(t) \, dt = 0,
\]

where \( t \) is time, \( y(t) \) is the normalized Y lead, and \( \tau \) is the time interval of P-wave activity.\(^{11}\) Once again the convention of a minus-plus normalized lead Z is observed.

The group averages and standard deviations of the rotatory angles A, B, and C, as obtained by the three methods of axis normalization, are listed in table 3. The last column of the table shows the length, width, and planarity (T for "thickness") of the normalized vector P loops in microvolt measurement. Length corresponds to the peak amplitude of the normalized X lead; width is the span between maximum and minimum in normalized Z lead; and planarity is the corresponding span in the Y lead. Histograms which depict the distribution of angular values and linear dimensions obtained by method I normalization are shown in figures 6 and 7. Table 3 also includes angles A and B (in essence, measures of azimuth and elevation) of the T\(_p\) and polar P vectors, with the corresponding histograms for T\(_p\), shown in figure 8. Figure 9 shows the distribution within the group of the spatial magnitude of T\(_p\) and P; that is, of the vectorialized area under these deflections in microvolt-second units of measure.

It is readily apparent from table 3 that the group behavior of normalized vector P loops is virtually independent of the method of normalization employed. A comparative study of angular and linear parameters as graphic displays lends further support to this observation since the frequency distributions for methods II and III normalization are similar to the method I distributions shown in figures 6 and 7. As will be discussed later, we have some preference for the first method.

Wave-Form Analysis

All unrotated and normalized scalar lead signals were written out on an incremental plotter and carefully inspected for wave form characteristics. A few examples of the plotted atrial wave forms are shown in figure 10. As had previously been our experience in dealing with fine-grained atrial deflections derived

\[\text{Circulation, Volume XXXIX, February 1969}\]
### Table 3

**Eulerian Angles and Quantitative Parameters of the Normalized Atrial Vectorcardiogram (Axial Lead System)**

<table>
<thead>
<tr>
<th>Method of normalization</th>
<th>Vector</th>
<th>Angles of Rotation and std (°)</th>
<th>Length (L), Width (W), Planarity (T) of Atrial Loop (µV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>P</td>
<td>A: -10.2 ± 37.0, B: 54.9 ± 23.5, C: 38.1 ± 38.0</td>
<td>L = 130.7 ± 37.6, W = 98.2 ± 25.8, T = 37.9 ± 15.0</td>
</tr>
<tr>
<td></td>
<td>Tp</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Polar P</td>
<td>A: -51.5 ± 27.8, B: -21.0 ± 21.3</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>P</td>
<td>A: -11.7 ± 39.3, B: 54.5 ± 25.7, C: 36.1 ± 39.6</td>
<td>L = 132.0 ± 37.5, W = 95.0 ± 24.9, T = 36.5 ± 14.2</td>
</tr>
<tr>
<td>III</td>
<td>P</td>
<td>A: -10.2 ± 37.0, B: 54.9 ± 23.5, C: 32.6 ± 37.1</td>
<td>L = 130.7 ± 37.6, W = 96.6 ± 26.4, T = 42.0 ± 17.7</td>
</tr>
</tbody>
</table>

**Figure 6**

Histogram depicting the Eulerian angles of rotation which normalize P-wave orientation according to the criteria of method I described in the text. Angle A is simply the negative of the azimuth of the vectoralized P-wave area, and angle B is identically this vector's angle of latitude. After these first two rotations have been executed about axes Y and Z, respectively, a final rotation of the vector forces (angle C about the X axis) produces a minimum-energy complex in the normalized Y lead and a “minus-plus” diphasic P wave in the Z lead. The group-average results of this method are found to compare very closely with those obtained from two other methods of axis normalization which have been described in the literature.
NORMAL ATRIAL VECTORCARDIOGRAM

P-WAVE LENGTH, WIDTH & PLANARITY

METHOD I

<table>
<thead>
<tr>
<th>NUMBER OF SUBJECTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>WIDTH</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LENGTH</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>PLANARITY</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>(μV)</th>
</tr>
</thead>
</table>

Figure 7

Histogram depicting quantitative parameters of the vectorcardiographic P loop in 106 healthy subjects, normalized by method I. Length is determined as the maximum amplitude of deflection in the normalized X lead. Width and planarity are the span between maximum and minimum deflection in the normalized Z and Y leads, respectively.

from bipolar extremity leads, we again found that the normal P wave has a richly notched and slurred configuration. Particular attention was devoted to P waves of maximal area, that is, to normalized X leads obtained by method I rotation, in the hope that some sort of clear and consistent separation into a right and left atrial segment would emerge. Unfortunately no such demarcation into two distinct phases was discovered, and apparently the only consistent morphological feature of the maximized P wave is its basic monotonic pattern.

The question of P-wave configuration was pursued further by the technique of pyramided factor analysis. The time base of each of the maximized P waves was normalized to 62 samples by simple linear interpolation. Six groups of 13 and two groups of 14 wave forms were each resolved into a set of uncorrelated base signals by the method of factor analysis. The first seven factors of each group were combined with those of another group to form four new groups of 14 each. These were again factored and regrouped until a final output of seven orthogonal signals was obtained which represented the distillation of wave form content of the original 106 signals. These final principal factor wave forms are illustrated in figure 11 and their relative magnitudes are listed in the second and third columns of table 4.

The base signals were next placed on an equal footing by scaling them to the same root-mean-square (RMS) value, and then fitted to each of the maximized P waves by a least-squares method. In this scheme of things the P wave may be treated as a unit vector in multi-dimensional signal space, with the contribution of each principal factor wave form being analogous to an individual component of the signal vector. The magnitude of difference between the unit signal vector and the vector sum of the seven basic components represents residual error. It is this quantity which is minimized in the least-squares method of synthesizing an input signal from principal factor wave forms. The remaining columns of table 4 summarize our results in applying this procedure to the whole set of normalized P waves.

Factor analysis confirms objectively the subjective impression that the P-wave configuration is richly varied and does not seem to fall into any set groups of patterns. One concludes from table 4 that each of the maximal area P waves contains a strong factor 1 component. However, bearing in mind that all the scale-normalized principal factor wave
forms have about the same zero-to-peak amplitude, it is also evident from table 4 that the basic prime-order pattern must be strikingly modified by the superposition of higher order components. Rather interestingly, and probably not fortuitously, the root-mean-square values of relative weights of factors 2 through 7 (column 2) are very nearly the same as the standard deviation (column 5) of the coefficients which represent their individual contributions to the entire group of P-wave morphologies. The implication of this observation is that, for each of the principal factors above the first order illustrated in figure 11, approximately 32% of the group contains as much of, or more than, the waveform shown in the illustration, with reversal of polarity in about half of these.

An attempt was also made to classify P waves by grouping them according to the relative separation of their vectors in signal space. The method is similar to Stark and co-workers’ separation of waveform families by multiple adaptive filtering but with the groups of waveform first resolved into a set of mutually independent base signals. With a fairly liberal tolerance limit, two major subgroups were formed which contained all but...
Table 4
Principal Factor Analysis of P-Wave Configuration

<table>
<thead>
<tr>
<th>Factor</th>
<th>Root-mean-square value</th>
<th>Peak-to-peak value</th>
<th>Relative weight (% of factor 1)</th>
<th>Content of equal-weight factors in maximal amplitude P waves (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>1</td>
<td>100.00</td>
<td>100.00</td>
<td>97.620</td>
<td>2.63</td>
</tr>
<tr>
<td>2</td>
<td>17.31</td>
<td>31.05</td>
<td>0.094</td>
<td>17.00</td>
</tr>
<tr>
<td>3</td>
<td>9.86</td>
<td>20.46</td>
<td>0.055</td>
<td>9.69</td>
</tr>
<tr>
<td>4</td>
<td>7.20</td>
<td>16.10</td>
<td>-0.004</td>
<td>7.08</td>
</tr>
<tr>
<td>5</td>
<td>5.72</td>
<td>12.04</td>
<td>0.014</td>
<td>5.66</td>
</tr>
<tr>
<td>6</td>
<td>4.45</td>
<td>9.48</td>
<td>-0.020</td>
<td>4.47</td>
</tr>
<tr>
<td>7</td>
<td>3.19</td>
<td>6.09</td>
<td>0.033</td>
<td>3.60</td>
</tr>
<tr>
<td>Residual</td>
<td></td>
<td></td>
<td>5.145</td>
<td>1.80</td>
</tr>
</tbody>
</table>

Discussion

The manner in which signal averaging improves the quality of P waves in the extremity leads has been previously described. Random noise content is reduced, the base line is stabilized to a reliable level of reference, and low-frequency distortion may be corrected if necessary. Applying the same signal-processing methods to orthogonal leads improves their quality and resolution to such an extent that it is now possible to extract almost as much quantitative information from the atrial vector forces as has hitherto been available only in dealing with the ventricular forces. Even the process of atrial repolarization, although partially obscured by ventricular activation, is susceptible to a limited amount of apparently meaningful quantitative treatment. Thus a variety of atrial vectorcardiographic entities, such as spatial P and T vectors, atrial eigenvectors, and polar P vectors, emerge from this study in well-defined and precisely quantifiable form.

This report describes three somewhat different methods for normalizing the atrial vector loop by re-orienting it in space according to uniquely defined criteria. Judging from the great similarity of results obtained from the three methods, it hardly seems to matter which one is selected for use. The rotational criteria, or "end points," of each method are described in the text. Because an iterative...
Several examples of atrial complexes as obtained with the axial-system set of corrected leads. The methods of signal enhancement and write-out which were employed are described in the text. The scale marks correspond to the millimeter scale of conventional records; that is, 0.04 sec per interval of abscissa, and 100 μV per interval of ordinate. There has been no rotation of axes in the two examples on the left. Immediately to the right of each of these the same time-functions are shown after normalization of the P loop by method I described in the text. The remaining examples are write-outs from another two subjects after P-wave normalization. These examples show the relatively rich notching which is almost invariably seen in the P waves of healthy subjects. In most of the records no division occurred which might be interpreted as separating the P-wave configuration into right and left atrial portions.

The methods of reporting the parameters of normalization appears to be of considerable importance. The procedure is basically a rotational transformation from one set of orthogonal axes to another. It could therefore be described by three sets of direction cosines (nine numbers) or by the azimuths and polar angles.
activation, it is tempting to try to divide the P wave into two or more phases which are specifically related to the anatomic sequence of the process. From our own experience, however, we are unable to find any stable characteristic of the normal P wave which morphologically divides it into “right atrial” and “left atrial” portions. This negative finding is at variance with the reported separation of the normal P wave in lead II by a distinctive notch in its midportion.1 Our findings also fail to agree with the observation that projection of the atrial loop in the horizontal plane, particularly when recorded by the timed technique,* characteristically separates into two distinctive convolutions.16

To assure ourselves further that this phenomenon did not occur in axial-system leads, we applied the technique of timed vectorcardiography to the atrial loops in their most open projection, with the additional feature that the direction of original translocation within the plane could be varied at will. Of the total group there were 13 with no visible separation of the loop into two phases, 44 with slight, 41 with moderate, and eight with marked separation. None showed division into completely separate subloops.

In a further attempt to discover some sort of configurational divider between the right and left phases of atrial activation, time-based scalar plots were made of spatial vector magnitude and of spatial vector velocity. Neither type of display revealed evidence of the sought-for information.

It has previously been difficult, and in some respects impossible, to examine and analyze the atrial electrocardiogram in fine detail. Many of the past difficulties have been resolved by modern technological advances which now place such kinds of study within the realm of feasibility. It has been the purpose of this report to describe the electrophysiological function of normal atria as man-

*The recording paper is moved uniformly leftward during registration, causing rightward translation of the origin at a constant rate. The single cube lead system was used in the study referred to.
manifested in a set of simultaneously recorded orthogonal leads. Since the distribution of atrial potential over the body surface is qualitatively dipolar,\textsuperscript{17} at least in normals, it may be that virtually all information which is available from external leading is in fact contained within the orthogonal set. Some items of information derived from the study are of immediate clinical relevance. Other information will have to be held in reserve, to be used as standards of comparison after a sufficient number of abnormal records has been accumulated.

References

Spatial Parameters and Shape Factors of the Normal Atrial Vectorcardiogram and Its Scalar Components

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