On the Path of the Excitation Wave in Atrial Flutter

By George R. Stibitz, Ph.D., and David A. Rytand, M.D.

SUMMARY
Available data on the arrival times of excitation at various central and peripheral atrial sites during atrial flutter (in a number of dogs and in one patient) have been examined in relation to families of involutes of an arbitrary central obstacle. The degree of fit seems to be satisfactory and is superior to that made for the same data by central-centrifugal (mother-daughter) waves. The latter appear to be an erroneous corollary of the circus movement hypothesis. In contrast, the concept that successive wave fronts of an entrapped circuit wave in atrial flutter may be described by a family of involutes has an appropriate physiological basis in the construction of Huygens, as first suggested by Wiener and Rosenblueth.

Additional Indexing Words:
Involutes of obstacle
Circus movement hypothesis
Mother-daughter waves
Huygens' principle
Entrapped circuit wave
Central-centrifugal waves

A STRIKING WEAKNESS of the circus movement hypothesis in atrial flutter, as ordinarily formulated, relates to the mother wave-daughter wave (or central and centrifugal) concept introduced by Lewis and associates for intact atria subsequent to clear demonstrations of entrapped circuit waves in more narrow rings of tissue. Rothberger and Schert and Schott, in objecting to the circus movement hypothesis, especially objected to the mother wave corollary. As stated by Schert and Schott, "Rothberger's objection . . . was that the recorded waves . . . were most unlikely to have been due to the excitation of the tissue in the path of the circulating wave, involving thin muscle bundles, but were due to the activation of the mass of auricular muscle. Rothberger contended, therefore, that it was not permissible to draw any conclusions about the course of a 'mother' wave from deflections which are due to the whole of the auricular musculature." Schert recently restated this reasonable objection.

Another recent review also noted the probable error of the "mother wave" in any narrow pathway concept, regarded this as irrelevant to the main problem of circus movement, and rather vaguely suggested "that relatively large atrial masses are activated in sequence during flutter as if from a leading excitation wave, the central portion of which may occupy a relatively narrow path including specialized tissues near the venae cavae." In response, one of us (G.R.S.) suggested that the entrapped wave of flutter could travel about a central obstacle as if following a family of involutes of that obstacle, thus independently considering an earlier but ignored concept of Wiener and Rosenblueth. The purpose of this study is to test such an hypothesis, as an alternative to the mother-daughter or central-centrifugal wave concept, against existing data.
Methods

Data concerning times of activation at both central and peripheral atrial sites during experimental flutter in the dog were reported by Lewis and associates\(^1\) (their figures 2, 3, 7, and 11), Kimura and associates\(^7\) (their figures 4b, 5, and 6). Takayasu and associates\(^8\) (their figures 7, 8, and 9), Ikuta\(^9\) (his figures 16, 20, 24, and 26), and Hayden and associates\(^10\) (their figure 6). For man, we have only the data from figure 16, especially sites C and D, of another paper.\(^5\) In most of these reports, the data were presented as maps of timing related to drawings of the atria; in some, central and centrifugal arrows showed the authors’ concepts of direction of excitation.

Data of the previous paragraph were photographed and projected from slides to superimpose, as well as possible, the obstacle of the venae cavae onto that of an arbitrary oval obstacle. Involutes of that obstacle were calculated and drawn to represent 0.02-sec intervals for the dog data and 0.05-sec intervals for those of man. For simplicity, it was assumed that velocity of propagation is constant and that all of the canine flutter cycles were either 0.12 or 0.16 sec in duration.

Huygens’ Construction: Involutes

Wiener and Rosenblueth\(^6\) have pointed out that the progression of waves in unspecialized cardiac muscle can be determined by a geometric construction devised in 1690 by Huygens for light waves. Even though the mechanism which justifies the use of the construction is entirely different in the two cases, the constructions are identical. Both applications rest upon a statement called “Huygens’ principle,” which asserts that each point along a wave front at any instant may be considered as a source of a new wave, and that the co-action of these new waves produces the wave front for a slightly later instant. Wiener and Rosenblueth gave a rigorous and fairly detailed derivation of Huygens’ construction for cardiac waves. Here we present a brief summary of the application.

We assume that when a point on the surface of a muscle sheet is stimulated, a wave of response travels out from that point at a velocity, \(C\), uniformly in all directions. If the first point, \(P\), is stimulated at time \(t_0\), then at a slightly later time, \(t_1\), there will exist a wave front which has the form of a circle of radius \(C(t_1 - t_0)\), provided there are no obstacles to propagation within this circle. The upper circle of figure 1 is such a wave front. Any point, \(P_1\), within the circle of radius \(C(t_1 - t_0)\) will have been stimulated at an instant prior to \(t_1\).

Next, let two points, \(P\) and \(P_2\), be stimulated at time \(t_0\). From each, a wave will travel outward as before. However, any point such as \(P_3\), which would have been stimulated at time \(t_1\) had \(P\) alone been stimulated at \(t_0\), will respond prior to \(t_1\) as a consequence of propagation from \(P_2\). It is assumed that there is a characteristic refractory period for the muscle such that \(P_3\) cannot respond a second time by propagation from \(P\). Therefore, at time \(t_1\) the wave front will not consist of two complete circles, but only of that part which is the locus of points on the circles, no point lying within either circle.

Next consider a wave front, \(W\), in figure 2. Relying on Huygens’ principle, we may think of each point on the wave front or locus of points stimulated at time \(t_0\) as a source of a spreading wave, which at time \(t_1\) has the form of a circle of radius \(C(t_1 - t_0)\). It is clear that except at singular points like the endpoints of the wave \(W\), if any, the locus of points stimulated at time \(t_1\) is the envelope of the family of circles of this radius, drawn with all the points on \(W\) as centers.

Several geometric properties of curves constructed by Huygens’ procedure are of interest:

First, we note that the new wave front is parallel to the old one, in the sense that they are everywhere equally spaced. That is, for any given point on one front, the least distance to points on the other is equal to \(C(t_1 - t_0)\).

Next, if a line is drawn perpendicularly to one wave front, it is also perpendicular to the other. Such lines may be thought of as “rays” along which the wave motion is being propagated at constant velocity.

We have thus far considered wave propagation in a surface without obstacles. Now we turn to the application of Huygens’ construction to the case of propagation in the neighborhood of an obstacle, such as \(O\) in figure 3. Muscle wave propagation is simpler, here, than light or sound wave propagation is, since waves of the former which impinge on an obstacle are destroyed without reflection because of the refractory state found in muscle cells. As wave front \(W\) in figure 3 touches obstacle \(O\), at time \(t_1\), say, those points along the new wave front, \(W_1\), that are in contact with the obstacle do not act as sources and can be ignored in the construction of further fronts.

Points on \(W_1\) that lie beyond the obstacle, however, are sources for further wave propagation. In particular, those near the edge of the obstacle do emit spreading waves, portions of which are thus propagated around the edge of the obstacle. If we define the distance of any point on \(W_2\) from front \(W\) as the length of that path starting at the given point on \(W\) and reaching wave front \(W_2\) without crossing the obstacle, and select that path for which the length is least,
then wave front $W_2$ is the locus of points at distance $C(t_2 - t_0)$ from wave front $W$.

Finally, consider the propagation of a wave from a point $P$ in contact with an obstacle, as in figure 4. At time $t_1$, the wave front takes the form shown as curve $I_1$ in figure 4. All points on curve $I_1$ are at the distance $C(t_1 - t_0)$ from point $P$, in the sense defined above. Similarly, at time $t_2$ the wave front has the form shown as curve $I_2$, and at time $t_3$ it is at $I_3$, and so on. Curves con-
Canine counterclockwise flutter, after figure 2 of Lewis and associates.\(^1\) Cycle length, 0.157 sec.

Canine counterclockwise flutter, after figure 3 of Lewis and associates.\(^1\) Cycle length, 0.171 sec.

Canine counterclockwise flutter, after figure 4b of Kimura and associates.\(^7\) Cycle length, 0.130 sec.

Canine counterclockwise flutter, after figure 9 of Takayasu and associates.\(^8\) Cycle length, 0.188 sec.

Involutes for an obstacle of arbitrary shape, such as that in figure 4, may be drawn physically by attaching a thread at point P, and scribing curves I\(_1\), etc., with various lengths of thread. The thread length for curve I\(_2\), for example, is C(t\(_2\) - t\(_0\)), and in the process of construction is wrapped around the periphery of the obstacle. It will be noted that at each position of the thread during the unwrapping process, the thread lies along the shortest path or geodesic between

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Figure 9
Human counterclockwise flutter, after data of figure 16 of Rytand. Cycle length, 0.29 sec.

Figure 10
Canine clockwise flutter, after figure 7 of Lewis and associates. Cycle length, 0.128 sec.

Figure 11
Canine clockwise flutter, after figure 11 of Lewis and associates. Cycle length, 0.160 sec.
its ends. This path therefore lies along a "ray" for the wave motion. In the case of the waves emitted from a point near the obstacle (fig. 4), the rays are tangent to the periphery of the obstacle. Involute representing successive wave fronts may be calculated for, say, an elliptical obstacle by calculating first the time required for the wave to pass along that periphery, or equivalently, the distance for such passage, and then marking off equally spaced wave fronts along the tangents to the peripheral points. Involutes for pulses at 0.02-sec intervals have been calculated with the help of the Dartmouth Computer, and used as the basis for the accompanying graphic representation of waves in the atrium.

It should be noted that waves in the heart take place in a surface that is not plane, and in fact cannot be drawn on a plane without distortion of distance relations. Furthermore, it appears that the wave velocity may be variable rather than uniform over the atrial surfaces. Finally, it should be emphasized that the presented curves are drawn as involutes about an elliptical obstacle which is at best only a crude approximation to the actual obstacles in the atria. For all these reasons, the figures shown herein represent only a first step toward an analysis of atrial flutter waves.

**Results**

Figures 5 to 9 present the results for wave progression as in the common type of flutter in man (counterclockwise in left sagittal, frontal, and horizontal planes, cranial in the left atrium, and caudal in the right atrium). Figures 10 to 13 show results for excitation in the opposite direction, rarely recorded in man. In figures 5 and 6, the right atrium is seen from the right; in figures 7 to 13, both atria are viewed as flattened and from the rear.

**Discussion**

The determination of activation times is sometimes relatively inexact, and there may have been even larger errors in locating their sites and in transferring these from a three-dimensional tissue to a plane surface. Furthermore, the actual velocities of wave fronts are unknown and may be quite variable; in particular, velocity often seems greater in the left atrium although such estimates should be regarded with reservations. Our inability to assign times to the involutes in the one human case (fig. 9) may be related to local variations in wave-front velocity. Despite such difficulties, it appears that the curves for families of involutes agree with the reported determinations remarkably well and represent wave fronts with fewer discrepancies than those which exist between the data and the central-centrifugal wave concept.

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PATH OF EXCITATION WAVE

Some of the findings are particularly noteworthy. Synchronous excitation of the two atrial appendages in counterclockwise flutter discussed elsewhere⁵,¹⁰ is nicely accounted for by the wave fronts when these are regarded as involutes (fig. 7). Central and peripheral sites of nearly simultaneous activation often fall within the area bounded by two adjacent involutes (figs. 9, 10, 12, and 13). A number of the centrifugal arrows, especially in the right atrium, drawn by earlier workers (that is, figs. 11 and 13) are nearly perpendicular to the wave fronts as indicated by involutes of the arbitrary obstacle; thus, they are equivalent to rays (see figs. 2 to 4).

For brevity, we have omitted the data from certain sources mentioned earlier.⁸⁻¹⁰ Their inclusion would not have altered the results.

It seems possible that specialized internodal tissues near the caval obstacle may be involved in the pathway of atrial flutter, but this remains speculative.⁵ The thesis of involutes presented here is compatible with such participation but does not depend upon it.

Huygens’ principle appears to afford a satisfactory physiological basis for believing that excitation in atrial flutter, shown elsewhere to behave as an entrapped circuit wave,⁵ proceeds about a central obstacle in such a way as to permit the representation of successive wave fronts by a family of involutes of that obstacle. In contrast, there is no apparent physiological basis for the mother-daughter wave concept.

Acknowledgment

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References

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