Quantitation of Precordial Movement

II. Mitral Regurgitation

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SUMMARY

Precordial displacement records of 39 patients with mitral regurgitation have been analyzed using a new method of quantitation. Three groups of patients were studied, and the data from each group have been compared in a number of different ways to those obtained from a control group. The optimal method of discrimination is a graphic representation involving the two variables of the height of the outward movement in systole and the height of the rapid-filling wave. This method allows discrimination between normal persons and patients with mitral regurgitation in a high proportion of cases.

It is postulated that the increase of systolic outward movement in patients with mitral regurgitation is related to an increase in stroke volume of the left ventricle, and the increase in the height or the slope of the rapid-filling wave represents increase in flow and rate of flow into the ventricle in early diastole. The importance of consideration of the functional status of the ventricle in the assessment of precordial movement is stressed.

Additional Indexing Words:
Systolic outward movement
"a" wave
Cardiogram, apex
Rapid-filling wave
Stroke volume

The assessment of severity of mitral regurgitation has been carried out in various ways: examination of the direct or indirect left atrial pressure pulse with particular emphasis on mean pressure, height of v wave, and rate of y descent; indicator-dye dilution studies involving calculation of regurgitant flow from the left ventricle to the left atrium; the use of angiocardiography, originally with a rapid series of films following injection of contrast material into the left ventricle, and more recently by examination of the left ventricle by cineangiography; and the calculation of regurgitant volume by means of an x-ray technique which allows estimation of ventricular volumes in systole and diastole. None of these methods is ideal; all involve performing cardiac catheterization.

Various investigators have shown in a qualitative way that patients with mitral regurgitation have an abnormality of the precordial displacement record taken over the cardiac apex. This abnormality is usually described as an exaggeration of the rapid-filling wave in early diastole. A method for quantitating displacement records, previously described, has been used to study 39 patients with mitral regurgitation. Measurements obtained are analyzed to permit discrimination from records of a normal control group. The establishment of such quantitative analysis would be a preliminary step in the development of the use of external displacement records to assess the severity of mitral regurgitation.

Methods

All patients seen in this hospital with a clinical diagnosis of mitral regurgitation in a 1-year period were surveyed. Patients who had aortic valve disease in addition to dominant mitral...
regurgitation were excluded but not those who were thought or shown to have trivial mitral stenosis. A total of 39 patients was studied. These were divided into three groups.

Group A (Cath in figures), comprised 14 patients in whom cardiac catheterization was carried out, and in some instances, subsequent surgery. Patients with left ventricular disease with incidental mitral regurgitation were included and are referred to later.

Group B₁ (XR + in figures) consisted of 16 patients who attended the cardiac out-patient clinic during the 1-year period. These patients had mitral valve disease with dominant regurgitation and no other valve was judged to be clinically abnormal. In some patients mitral stenosis was also present, but was not thought to be important. This group differed from group B₂ in that all patients exhibited radiological abnormality (namely, left atrial enlargement, or increase in cardiac size, or both, and pulmonary vascular abnormalities).

Group B₂ (XR - in figures) consisted of nine patients collected in the same manner as those in group B₁. The diagnosis in every case was made clinically, usually simply on the basis of a pansystolic murmur at the apex and an occasional third heart sound. There was no radiological abnormality.

The distribution of all these patients according to age, sex, and weight is shown in table 1. A record of apical displacement could be made easily in every case by contrast with the normal control group which included an 18% failure rate.

The methods of recording and of making the various measurements of the a wave, outward movement (OM), and rapid-filling wave (RFW), and of calculating the ratio of the height of RFW/ duration were identical to those described for the normal control group in part I of this study.

Results

Individual results for each of the patients in the three groups are shown in table 2. An example of a record of a patient with mitral regurgitation is shown in figure 1. Analysis of the results in the three groups is shown in table 3. When these results are compared with those for the control group, the striking abnormalities are in the height of the OM in all three groups: group A, 54x ± 26 (SD); group B₁, 61x ± 31; group B₂, 48x ± 23; and the normal group, 25x ± 7. At the same time, the RFW is also increased clearly in groups A and B₁ (22x ± 17 for group A, and 18x ± 7 for group B₁) by contrast with the normal

Figure 1

Displacement record of patient with mitral regurgitation. Phonocardiograms taken simultaneously in pulmonary area (PA) and at left sternal edge (LSE), at high frequency (HF). Simultaneous electrocardiogram (lead II). Heights of a wave, systolic outward movement (OM), and rapid-filling wave (RFW) were measured as shown. Height of standard impulse (X) was measured from record made at same sensitivity setting as shown. Calculation of values for a, OM, and RFW in terms of X is shown.

<p>| Table 1 |
|---|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>Group*</th>
<th>Number of</th>
<th>Age, range (yr)</th>
<th>Weight, range (lb)</th>
<th>Total number</th>
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<td></td>
</tr>
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<td>6</td>
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<tr>
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<td>40-166</td>
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<td>50-140</td>
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<td>3-54</td>
<td>40-190</td>
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</table>

*See text for subdivision of patients into groups.

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group (7x ± 3). The abnormality of RFW is less striking for group B2: 11x ± 4. The a wave of the patients with mitral regurgitation is not statistically nor practically different from that of the control group, but there are occasional, individual striking differences, and this feature has been discussed. The presence of atrial fibrillation in five patients excluded measurement of the a wave, and in three other patients its precise height was impossible to define.

A plot of all the patients and the normal control group using the two variables, OM and RFW, as axes is shown in figure 2. A

### Table 2

<table>
<thead>
<tr>
<th>Patient</th>
<th>Age (yr)</th>
<th>Wt (lb)</th>
<th>Sex</th>
<th>a*</th>
<th>OM*</th>
<th>Height*</th>
<th>RFW</th>
<th>Duration (sec)</th>
<th>Heart rate</th>
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<td>4x</td>
<td>0.06</td>
<td>75</td>
<td></td>
</tr>
</tbody>
</table>

*Values for heights of a wave, OM, and RFW are in terms of x. In some instances the a wave could not be accurately defined; this is represented as —, or atrial fibrillation (AF) when present.

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tolerance region which encompasses 95% of the normal population with 95% confidence has been drawn on this graph. The method of calculating this region, based on the work of John, is described in detail in the appendix. All but three of the normal group fall within this region, while all but 10 of the patients with mitral regurgitation fall outside it.

More conventional methods of plotting the data for the control group and for the patients studied are shown in figures 3 to 6, where the individual values for a, OM, RFW, and RFW/duration are shown, and the upper tolerance limit for 95% of the normal population is marked. With the exception of the a wave, all these graphs show the discrimination between the group of patients with mitral regurgitation and the normal group. The bivariate tolerance region makes fewer errors of classification than any of the tolerance limits based on a single characteristic.

**Discussion**

The results described confirm the belief that it is possible to discriminate in a large proportion of cases patients with mitral regurgitation from normal persons by a quantitative method of analysis of apical precordial displacement records. Qualitative

---

**Table 3**

Results According to Groups for a Wave, OM, RFW, and Slope of RFW: Mean Values and Standard Deviations

<table>
<thead>
<tr>
<th>Group</th>
<th>Mean ± SD</th>
<th>Group</th>
<th>Mean ± SD</th>
<th>Group</th>
<th>Mean ± SD</th>
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<tbody>
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<td>a wave*</td>
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<td>OM*</td>
<td>54x 26</td>
<td>RFW*</td>
<td>22x 17</td>
</tr>
<tr>
<td>RFW/ (duration × 100)</td>
<td>3.9 2.8</td>
<td>RFW/ (duration × 100)</td>
<td>3.4 1.8</td>
<td>RFW/ (duration × 100)</td>
<td>1.8 0.7</td>
</tr>
</tbody>
</table>

*Heights of recordings are expressed in terms of a constant signal, x. For details see Part I.6

---

**Figure 2**

Graph showing all patients with mitral regurgitation and all normal controls using the heights of the systolic outward movement (OM) and rapid-filling wave (RFW) as axes. Subdivision of patients with mitral regurgitation into catheterized (CATH [group A in text]), cardiovascular chest x-ray abnormalities (XR+ [group B1 in text]), and normal chest x-ray (XR− [group B2 in text]), is made by symbols as shown. The tolerance region encompassing 95% of the normal population with 95% confidence is drawn. Three normal patients fall outside the region, while 10 patients with mitral regurgitation fall inside it.

**Figure 3**

Values for a wave height are shown for all normal controls and patients with mitral regurgitation. Upper tolerance limit and mean value for normals are shown. Symbols same as in figure 2.
analyses of such records of patients with mitral regurgitation have stressed the importance of an exaggeration in the relative height of the rapid-filling wave to the maximal height of the record, and in the angle formed between it and a horizontal line. These methods have not permitted absolute measurements of different aspects of a displacement record and consequently their use has not permitted recognition of cases in which both the rapid-filling wave and the systolic outward movement are exaggerated in the same ratio to the normal. This uniform exaggeration constitutes a hyperdynamic impulse, which is a characteristic feature of mitral regurgitation, as well as of other hemodynamic abnormalities of the cardiovascular system, for example, thyrotoxicosis. The method of quantitation described above enables a hyperdynamic impulse to be measured, and at the same time it will pick out selective abnormalities in the heights of the various waves individually.

**Systolic Outward Movement**

A striking feature of all the groups studied is the exaggeration in the height of the systolic outward movement. This may reflect the increase in volume handled by the left ventricle assuming that the intraventricular pressure and wall thickness remain normal. An uncomplicated case of chronic mitral regurgitation in which left ventricular function has not been significantly deranged is...
hemodynamically abnormal by virtue of the increased volume of blood handled by the ventricle. The patients in group B1 fall mostly into this category, and in most instances they show an increase both in OM and in RFW.

The shape of the systolic outward movement in the records of most of the patients with mitral regurgitation does not differ appreciably from that of the normal group. Once systolic outward movement has reached its peak shortly after the first heart sound, it falls away to a shoulder at the time of aortic valve closure and then falls precipitously to the trough which marks the opening of the mitral valve. This is in contrast to records of patients with aortic stenosis who often have an exaggeration of the height of the systolic outward movement, however with a convex configuration.

Rapid-Filling Wave

Both the height of the rapid-filling wave and its slope (that is, RFW/duration) are significantly increased in the majority of patients with mitral regurgitation. The timing of the rapid-filling wave corresponds with the period in early diastole of rapid ventricular filling. In mitral regurgitation both the volume of blood entering the dilated left ventricle and its rate of entry will be increased if the degree of valve abnormality is hemodynamically significant, and mitral valve obstruction is not an additional feature. Apart from the patients with left ventricular dysfunction referred to below, only one patient, N.D., known to have severe mitral regurgitation by findings at cineangiography and surgery failed to show exaggeration of the rapid-filling wave. The reason for this exception is uncertain.

Effect of Ventricular Function

The role of ventricular function in these recordings is important, and little emphasis has been laid on this aspect in previous accounts. From the small group of patients studied, it is impossible to draw any definite conclusions about it. The data obtained at cardiac catheterization would have to include data on such parameters as ventricular volume and pressure, cardiac performance at rest and in response to exercise or drugs, and data from other sophisticated tests of cardiac function before it is possible to make any definite statement. Such information was not available in these patients. Nevertheless, it is reasonable to speculate that, under conditions of severe ventricular dysfunction, the regurgitant volume through the mitral valve may become depressed as the ventricle loses its contractility, and only pressure changes occur rather than striking alterations in volume. If the rapid-filling wave reflects alteration of volume during early diastole, it will become unusually small in a case of severe mitral regurgitation in terms of the abnormal orifice size, given a ventricle which is neither able to eject nor to distend normally. A less incompetent valve with a functionally normal left ventricle may produce a large RFW.

There were two patients, C.W., and H.B., who had mild left ventricular disease, and in whom mitral regurgitation was trivial, as judged by left ventricular cineangiography. These patients are noteworthy as the
height of both OM and RFW are not increased, and their results would fall into a normal range. This is not surprising if the concept of alteration in intracardiac volume producing the most striking abnormalities in the precordial record is tenable.

Acute severe mitral regurgitation in which ventricular dilatation has not had time to occur may fail to be associated with an increased OM. This was the case in patient R.A. in group A. If ventricular dilatation has not taken place and the degree of mitral valve disruption is severe, the stroke volume of the left ventricle may not be significantly altered. Left atrial work will be increased in response to mitral regurgitation; however, the ventricle is already maximally filled at the time of atrial systole, and instead of producing a significant flow of blood into the ventricle, atrial work results in an elevation of left ventricular pressure, with an increased wave. This would be reflected in the displacement record as an abnormally large wave, and this was a feature exhibited by patient R.A.

One patient, J.H., had moderate left ventricular disease with elevation in end-diastolic pressure and only mild mitral regurgitation. The most significant abnormality in his precordial record was an increased wave reflecting the elevated end-diastolic pressure, while other parameters were not increased significantly.

Value of Normal Results

Of the patients who fall within the normal region shown in figure 2, four are patients in group B2 who have only an apical pansystolic murmur following an episode of rheumatic fever. In all cardiovascular respects they must be regarded as normal, and consequently it is not perhaps surprising that they fall into a normal region. Thus, the finding of a normal displacement record in a patient with a clinical impression of hemodynamic insignificance will help to reinforce that impression.

Appendix

The statistical problem encountered in this publication is that of distinguishing between a functionally normal population and an abnormal population composed of several subpopulations each presenting an abnormality whose precise nature can be determined only by further examination. The data collected from the abnormal group show a much greater dispersion than that from the normal, and the percentage contribution of each possible type of abnormality to the total population cannot be foretold. These two characteristics of the data prevent the application of the classical method of discriminatory analysis unless assumptions that appear unwarranted are made. Instead, the method of tolerance regions will be used to prescribe a boundary beyond which patients are classified as out of the normal range.

Method

John and Siotani have published methods for constructing a tolerance region based on a sample from a multivariate normal distribution. The methods proposed by these authors differ from each other and both are approximate. It is not known which of the two methods is more appropriate, but John's is easier to apply, and there is reason to believe that the degree of approximation involved in his method does not seriously upset the probability levels desired for most applications.

Briefly John's method is to define the region

\[(x - \bar{x})' V^{-1} (x - \bar{x}) < K, \quad (1)\]

where

\[x = \text{the (lpk) vector of measurements}, \]
\[\bar{x} = \text{the (lpk) vector of means of values of normal subjects based on n observations}, \]
\[V = \text{(pxp) dispersion matrix estimated from normal subjects}, \]
\[K \text{ is a constant determined by} \]

\[K = (n - 1) \frac{v_1}{v_2}; \quad (2)\]

where

\[v_1 = P - \text{the percentage point of a noncentral } \chi^2 \text{ variate with p degrees of freedom and noncentrality parameter } p/2n, \]
\[v_2 = (1 - \gamma) - \text{the percentage point of a central } \chi^2 \text{ variate with } (n - 1)p \text{ degrees of freedom,} \]

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\( P = \) proportion of the population included in the tolerance region, and \\
\( \gamma = \) probability of including at least the proportion \( P \).

All individuals who fall outside the region defined by equation (1) are classified as belonging to the abnormal population.

If \( p/2n \) is small, and it usually is, \( \nu_1 \) can be determined from the central \( \chi^2 \) - distribution. If \( p/2n \) is not small, and tables of the percentage points of a noncentral \( \chi^2 \) - distribution are not available, the approximation developed by Abdel-Aty\(^{14} \) is accurate enough for most purposes. His approximation is

\[
\chi^2_\alpha = \left[ \frac{2}{9} \left( 1 + \frac{b}{r} \right) \right] + \left[ \frac{1}{9} \left( 1 + \frac{b}{r} \right) \right] \frac{3}{r} 
\]

\[
(x-x')' V^{-1} (x-x) = (x_2 - 24.5, x_3 - 6.6) 
\]

\[
0.0346 x_2^2 - 0.9871 x_2 + 0.0872 x_3 
\]

where

\( \chi^2_\alpha = \alpha - \) the percentage point of the non-central \( \chi^2 \) - distribution,

\( Z_\alpha = \alpha - \) the percentage point of the normal distribution,

\( r = \frac{p(2n+1)}{2n} \), and

\( b = \frac{1}{2n+1} \)

**Results of the Analysis**

The means and dispersion matrix of the four variables estimated from the 37 normal subjects in whom records could be made are shown in table 4. The dispersion matrix is carried out to several decimal places because its inverse will be needed in calculating the tolerance region.

A preliminary examination of the data showed that only two of the variables, OM and RFW (height), were discriminating between the normal and abnormal subjects. These two variates were combined in subsequent analysis to obtain the bivariate tolerance region shown in figure 2.

The ellipse shown in figure 2 was obtained as follows:

when \( p = 2 \), and the normal values of OM and RFW (height) are substituted in equation (1); it can be simplified to become

\[
\frac{x_2^2 - 24.5}{0.0346} - \frac{x_3 - 6.6}{0.05366} < K. 
\]

The expansion yields:

\[
0.1073 x_2 x_3 + 0.1926 x_2^2 + 11.8038 < K. 
\]

Utilizing equations (2) and (3) when \( p = 2 \) and \( n = 37 \) yields \( K = 8.1 \). After setting equation (4) equal to 8.1, various values for \( x_2 \) have been substituted, and the resulting quadratic equation has been solved for \( x_3 \). This has given the values necessary for plotting the ellipse.

When more than two variables are used to form the tolerance region, the graphic representation shown in figure 2 is no longer possible. Tolerance regions can still be computed, but at the expense of a less intuitive representation. In the bivariate case, the more general representation amounts to evaluating equation (1) at the values observed on a subject in question and observing whether the computed value is greater than the critical value of \( K \).

**Table 4**

*Means and Variances Estimated from Normal Subjects*

<table>
<thead>
<tr>
<th></th>
<th>( x_1 = a )</th>
<th>( x_2 = \text{OM} )</th>
<th>( x_3 = \text{RFW (height)} )</th>
<th>( x_4 = \text{RFW/duration} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Means</td>
<td>2.9</td>
<td>24.5</td>
<td>6.6</td>
<td>0.07</td>
</tr>
<tr>
<td>Dispersion matrix</td>
<td>2.34680</td>
<td>5.11460</td>
<td>3.35022</td>
<td>-0.00046</td>
</tr>
<tr>
<td>( V )</td>
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<td>14.18036</td>
<td>9.14392</td>
<td>-0.00572</td>
</tr>
</tbody>
</table>

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References


Critique on Critiques

The critical state of the criticial review grows from our constant forgetfulness of all this. The young scientist is taught carefully and methodically to be a quarryman or a brickleyer. He learns to use his tools well but not to enlarge his perspective, develop his critical powers, or enhance his skill in communication. The older scientist is too often overwhelmed by detail, or forced by the competition of the professional game to stick to the processes of “original research” and “training.” The vastness of the scientific literature makes the search for general comprehension and perception of new relationships and possibilities every day more arduous. The editor of the critical review journal finds each year a growing reluctance on the part of the best qualified scientists to devote the necessary time and energy to this task. Often it falls by default to the journeyman of modest talent, a compiler rather than critic and creator, who enriches the scientific literature with a fresh molehill in which later compilers may burrow.

Quantitation of Precordial Movement: II. Mitral Regurgitation
GEORGE C. SUTTON, ERNEST CRAIGE and JAMES E. GRIZZLE

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