Evaluation of Multipolar Effects in the High-Fidelity Standard Electrocardiogram by Means of Factor Analysis

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Many studies have rather clearly demonstrated the inadequacy of the simple dipolar concept to account for the total electrocardiographic information available on the body surface.\textsuperscript{1-6} For example, Okada, Langner, and Briller have attempted extraction of the dipolar component from the precordial leads by cancellation with synthetic dipolar leads derived from vectorcardiograms. An unaccounted-for residual was found.\textsuperscript{8} By means of closely spaced multiple lead recordings, subsequent workers have constructed isopotential maps demonstrating, not only dipolar pattern, but islands of apparent multipolar activity as well.\textsuperscript{1, 2, 4, 5}

Factor analysis provides still another means by which multipolar contribution may be detected.\textsuperscript{5, 7, 8} Mathematically, factor analysis compares each of a population of waveforms with every other waveform in the group, retains the several bits of unique information, and casts out the redundant information as it reappears in lead after lead. Previously, when this process of factor analysis was applied to total-surface electrocardiograms recorded from 153 to 160 closely spaced sites completely encompassing the thorax, at least seven or eight unique waveforms or principal factors were found. In brief, all of the nonredundant information present in the many QRS complexes could be expressed concisely in the form of seven or eight completely different waveforms.\textsuperscript{5}

Scher has indicated that finding not more than three principal factors when this process is applied to a population of QRS complexes is compatible with a dipolar generator. Such compatibility, however, is not proof of a dipolar generator, since a number of three-function generators other than a single dipole is possible.\textsuperscript{8} In addition, the emergence of more than three principal factors is not even compatible with a single dipole and immediately demands a more complex generator for explanation.

One knows at the outset that when the process of factor analysis is applied to the six frontal plane leads of the standard electrocardiogram, $I$, $II$, $III$, $V_R$, $V_L$, and $V_P$, that only two significant factors or waveforms should emerge. This follows from the fact that we are dealing with only three sampling points, the right arm, the left arm, and the left leg, which describe a plane and from the information in any two of these leads the information in a third may be predicted. In the contrasting case of the multiple-lead total-surface electrocardiogram obtained from man, it was necessary to employ eight principal factors of unique waveform to reconstruct the original QRS complexes. In this instance when the information in only the first three factors was employed in an effort to reconstruct the original QRS complexes, only 85 per cent of the information could be accounted for.\textsuperscript{5} Thus, at one end of the spectrum lies the frontal plane electrocardiogram, which may be represented by a two-function generator. At the other end of the spectrum there is conclusive evidence of a multipolar cardiac generator. The question then logically arises: How many factors or independent waveforms emerge when the

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Supported by grants from the Tennessee Heart Association and the American Heart Association, and by Grants HE-01362-12, 5-K6-HE-14, 032-03, HE-08861-01, and HE-09495 of the National Institutes of Health, U. S. Public Health Service.
standard electrocardiogram which consists of both the frontal plane leads and the six customary precordial leads is examined?

**Methods**

Simultaneously recorded high-frequency, high-fidelity leads I, V_F, V_1-6 were obtained from 36 healthy male subjects and photographed from the face of an 8-channel Tektronix oscilloscope. SVEC-III, McFee-Parungao axial and Frank vectorcardiographic leads were likewise obtained for each subject. A Grass C-4 camera was used for the photography with a film speed of 250 mm. per second. The services of McDonnell Aircraft Data Processing Division were utilized for the purpose of optical analog-to-digital conversion at a sampling frequency of 390 bits per second. Of the frontal plane leads only I and V_F were used to represent the three sampling points comprising the six limb leads, thus avoiding some of the obvious redundancy in the standard 12-lead electrocardiogram. Five subjects were eliminated either because of previous history of cardiac abnormality or artifact. The remaining 31 sets of recordings were smoothed by numerical filtering to exclude frequencies above 39 cycles per second. They were then subjected to factor analysis by means of computer processing. Resynthesis of the original electrocardiograms was performed by successive utilization of each subject's final principal factors and again by the use of the subject's McFee-Parungao vectorcardiogram as previously described. The cumulative discrepancy in absolute instant-by-instant amplitude between the resynthesized and the original QRS waveform is expressed in

**Figure 1**

Above is seen an example of the raw data. Below is its digitized counterpart as obtained from IBM 407 printer.
FACTOR ANALYSIS OF ELECTROCARDIOGRAM

principal factors

of

limb plus chest leads

limb leads only

After the process of factor analysis has been applied to the limb leads only of one subject (right) and to the entire standard electrocardiogram of another subject (left), the digitized principal factors have been cast into waveform.

Results

In figure 1 may be seen an example of the raw data, above, and its digitized counterpart, below. Figure 2 contains examples of computed final principal factors of both the frontal plane electrocardiogram and the frontal plane plus the precordial leads (standard electrocardiogram). The gross similarity of factors 1 and 2 is apparent and, in spite of the fact that not only more leads but a third physical dimension was added in the form of the six precordial leads, factor 3 remained small, but possibly significant. In table 1 there is a numerical analysis of the results of the entire group of 31 subjects. Over-all reconstruction was fairly good after the information contained in only the first two principal factors was utilized; the average percentage of divergence at this point was only 10.2 per cent. But careful examination of the lead-by-lead calculation of divergence reveals that although divergence is quite low in the precordial leads after the utilization of only two factors, there is a considerable amount of unaccounted-for

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The per cent of divergence was estimated by dividing the root sum square of the instant-by-instant deviation of the derived waveform from the original, by the root sum square of the deviation of the original waveform from the baseline, and multiplying by 100. Thus if \( a \) = the amplitude of the original waveform at an instant and \( d \) = that of the derived waveform at the same instant, then the percentage of divergence =

\[
\frac{\sqrt{\sum (a - d)^2}}{\sqrt{\sum a^2}} \times 100
\]

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\[
\frac{\sqrt{\sum (a - d)^2}}{\sqrt{\sum a^2}} \times 100
\]
Table 1

Per Cent of QRS Unaccounted for after Resynthesis by Factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>Average for 31 Subjects</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>24.7</td>
</tr>
<tr>
<td>V_F</td>
<td>19.5</td>
</tr>
<tr>
<td>V_1</td>
<td>10.5</td>
</tr>
<tr>
<td>V_2</td>
<td>3.1</td>
</tr>
<tr>
<td>V_3</td>
<td>4.9</td>
</tr>
<tr>
<td>V_4</td>
<td>4.9</td>
</tr>
<tr>
<td>V_5</td>
<td>5.9</td>
</tr>
<tr>
<td>V_6</td>
<td>7.8</td>
</tr>
<tr>
<td>Averages</td>
<td>10.2</td>
</tr>
</tbody>
</table>

The per cent of QRS unaccounted for (divergence of the resynthesized QRS from the originally recorded QRS) after a factor-by-factor resynthesis is illustrated. This represents the average for all 31 subjects. The last line shows progressive reduction in percentage of divergence as each factor is added to reconstruct the entire electrocardiogram. Above, this reduction of divergence is presented in terms of individual leads. After the utilization of only two factors the precordial leads are reconstructed rather well. The bulk of the divergence after the utilization of three factors remain in leads I and V_F. See text.

Table 2

Per Cent of QRS Unaccounted for after Resynthesis by Factors 1 and 2

<table>
<thead>
<tr>
<th>Subject H. L.</th>
<th>V_1</th>
<th>V_2</th>
<th>V_3</th>
<th>V_4</th>
<th>V_5</th>
<th>V_6</th>
<th>Average Divergence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.4</td>
<td>2.2</td>
<td>2.5</td>
<td>3.0</td>
<td>6.9</td>
<td>4.5</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

The relative effectiveness of resynthesis is presented in terms of per cent of divergence resulting from factors one and two alone in the case of the precordial leads only (on the left) and of the precordial leads plus the limb leads (on the right). Note that most of the inadequacy of resynthesis may be accounted for in leads I and V_F. See text.

information in leads I and V_F. Divergence in the precordial leads is further reduced with the addition of the information in factor 3 to well within the limits of noise. By contrast, appreciable reduction in the percentage of divergence of the reconstructed leads I and V_F from the original did not appear until the addition of the information in the first four factors. A typical example of this consistent variation in factor content between limb and precordial leads may be seen in figure 3. With the utilization of the information in the first two principal factors, lead V_2 was reconstructed in this subject to within 2 per cent of the originally recorded QRS complex while a two-factor reconstruction of lead I led to a divergence of 18.3 per cent. For the same subject illustrated in figure 3, table 2 contains a comparison of the results of factoring of the precordial leads alone with the results of factoring of the complete 8-lead electrocardiogram. All of the precordial leads were reconstructed to within 3.6 per cent of the original waveforms by the utilization of only the first two principal factors. The standard electrocardiogram was reconstructed to within 6.6 per cent of its originally recorded form, but the bulk of error still resided in leads I and V_F and is quite significant.

Because the clinical counterpart to a dipole is the vectorcardiogram, we compared (as shown in figure 4) the relative efficiency of reconstruction of QRS complexes from a given subject's McFee-Parungao vectorcardiographic leads and from the first three principal factors obtained from this subject's standard electrocardiogram. We thought that this might be a way to explore the problem of whether the first three principal factors in a given subject represented simply dipolar or multipolar comp-
The effectiveness of resynthesis of precordial lead $V_2$ from information contained in only the first two principal factors is illustrated. Lead I shows considerably more divergence when only the first two factors are utilized. In both cases the bold line represents the originally recorded QRS complex while the broken line represents the resynthesized QRS complex. See text.

A comparison was made between the effectiveness of the McFee-Parungao vectorcardiographic system to resynthesize the original QRS with the ability of the first three principal factors of the given subject to perform the resynthesis. This might be considered a comparison of a dipolar resynthesis (the VCG system) and a possibly multipolar resynthesis as represented by the first three principal factors. The dipole was able to reconstruct $V_1$ with a divergence of 8.9 per cent and $V_6$ with a divergence of 25 per cent. When the first three principal factors were utilized, however, the divergence in both cases was in the range of only 3 per cent. This possibly indicates that the first three principal factors contain more than simply dipolar information.
only about 3 per cent in both of these leads. We interpreted the relative difference in information content between principal factors and vectorcardiographic leads as possibly indicating that these three factors contained more than simply dipolar information.

**Discussion**

How then can we rationalize some of the seeming paradoxes involved? It is generally conceded that proximity leads such as the precordial leads should be relatively multipolar sensitive. In previous studies done in this laboratory regarding the relative sensitivity of the various leads in sensing quadrupolar, octapolar, and higher order components, it was found that each of the precordial leads was considerably more sensitive to these components than were any of the six extremity leads. 

How can we account then for the fact that when the precordial leads were factored alone, and an effort was made to resynthesize them by only two factors, that an average divergence was so low? Likewise, when the precordial leads were factored along with the extremity leads, the percentage of divergence in them with two-factor reconstruction was still considerably lower than in the extremity leads. If we were to assume that a two-factor generator is equivalent to a vector or dipolar generator, one possible explanation may be suggested by the model of the human torso illustrated in figure 5A. The principal axes of a fixed location dipolar generator are

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**Figure 5**

A, left. Represents a possible explanation for the effectiveness of only the first two principal factors to reconstruct the precordial QRS complexes. In the center of the illustration is a vector loop actually constructed from a given subject. The arrows extending at right angles from the isoelectric point of the vector loop represent axes of the principal factors one and two as they might appear if they represented a dipolar cardiac generator. The vector loop is extended along its own plane to illustrate where the extended ecliptic intersects with the torso. The precordial electrode sites are illustrated by the black dots. Their fortuitous placement along the perimeter of the plane of the vector loop renders them particularly vulnerable to dipolar swamping effect. B, right. Suggests an alternative explanation for the effectiveness of factors one and two to reconstruct the QRS complexes of the precordial leads. If principal factors one and two represent two individual dipoles lying in the same plane without a common origin and this plane is extended in the same fashion as the plane of the vector loop to intersect with the torso, only two principal factors might be needed to account for the information in the precordial QRS complexes but in this instance the first two principal factors become a multipolar representation (double dipolar) rather than a simple dipolar representation of the cardiac generator.
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illustrated by the arrows, and the vector loop generator itself is represented by the QRS spatial vector loop. The first two principal factors derived from factor analysis of the multi-lead “total surface” electrocardiogram each have a simple dipolar pattern of distribution on the chest suggesting axes with such an orientation. The vector loop was projected onto the surface of the torso in the form of an ecliptic (i.e., the boundary where the plane of the major axes of the vector loop intersects with the body surface) and the anatomic sites of the precordial electrodes were superimposed. The fortuitous placement of the precordial electrode sites in the standard electrocardiogram tends to follow the perimeter of the projected vector loop. Thus in attempting to utilize factors 1 and 2 as components of a simple dipolar generator, we would be compelled to say that although these leads are extremely multipolar sensitive because of their proximity relationships, they are also highly vulnerable to being flooded by dipolar signal because of their orientation along the ecliptic of the projected vector loop. The approximation of an equivalent dipole or heart vector by the first two principal factors quickly proves unsatisfactory. For example, why do leads I and V6—known to be less multipolar sensitive than the precordial leads—show a great deal more error upon resynthesis with two or three factors alone than do the six precordial leads? As just suggested, the answer may lie in the fact that the precordial leads are relatively more subject to dipolar swamping effect and the extremity leads, having a lower signal-to-noise ratio, simply contain more unaccounted-for noise. A more likely explanation as shown in figure 5B is that factors 1 and 2 do not represent orthogonal components of a simple fixed dipole but behave as two distinct dipoles of separate location and therefore contain in them multipolar expression as well. Since in factoring the standard electrocardiogram we have six precordial leads which should be rather heavily laden with multipolar activity, it is not unreasonable to expect the first two principal factors also to be so influenced. It may simply be the lack of multipolar content in leads I and V6 when compared to the precordial leads that causes them to be harder to resynthesize by the first two principal factors obtained from factoring them along with the six precordial leads.

Our conclusion then is, while factor analysis may be extremely helpful in characterizing the equivalent cardiac generator in general terms as nondipolar as soon as more than three principal factors are found, this work may be considered something of a statement of the limitation of factor analysis as a means of specifically describing an equivalent generator. When only three points are examined and only two factors are required to reconstruct the information contained in such a limited population of waveforms, the dipolar generator proves the simplest explanation. In spite of this fact, as soon as more than three points are examined and a variable number of factors are extracted, one may not assume that the first two of these principal factors are purely dipolar. In fact, it must be assumed that they express nothing more than the most commonly occurring waveforms in the group, be those waveforms dipolar, multipolar, or a combination thereof.

Summary

The usefulness of factor analysis as a means of describing in both general and specific terms an equivalent cardiac generator has been explored. High-fidelity, high-frequency electrocardiograms and vectorcardiograms of 31 normal subjects have been analyzed. The information in the electrocardiogram was reduced by factor analysis to the minimum number of independent waveforms which could account for all of this information. Because the first three principal factors might or might not represent a simple dipole, the efficiency of these three principal factors to resynthesize the originally recorded electrocardiograms was compared with the efficiency of McFee vectorcardiographic leads to perform the same resynthesis. The first three principal factors contained more information than that contained in the vectorcardiographic leads, sug-
gesting that the factors contain more than simple dipolar information. This would in turn suggest that a more complex generator than the dipole must be employed to explain the surface potentials derived from the standard electrocardiogram. With only two principal factors the precordial leads were rather easily reconstructed but reconstruction of leads I and $V_P$ was considerably more difficult. Because the principal factors represent the most commonly occurring waveforms in the group, the less perfect resynthesis of leads I and $V_P$ with two factors alone may result from lack of detection of multipolar component in the limb leads which have less in common with the precordial leads than the precordial leads do with each other.

References


How Medicine Became a Science
In the early part of the nineteenth century clinical science celebrated several successes. Methods of examination had improved by the introduction of percussion, introduced by Auenbrugger in 1761, and of auscultation devised by Laennec in 1816. In addition the knowledge of morbid anatomy in England had been advanced by the publication of Matthew Baillie's Atlas. Medical teaching in London was at that time specially concentrated at Guy’s Hospital and it was there that the increased knowledge bore most fruit. The names of Bright, Addison and Hodgkin are sufficient to prove that accurate observation, both of the living and the dead, careful consideration of what one has observed, and confirmation by further observation of nature’s experiments on the human frame, may serve to produce real scientific advances. Clinical science conducted with minimal laboratory aid still retains its value.—ZACHARY COPE, KT. Some Famous General Practitioners and other Medical Historical Essays. London, Pitman Medical Publishing Co., Ltd., 1961, p. 189.
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Circulation. 1965;32:273-280
doi: 10.1161/01.CIR.32.2.273

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