Use of Bayes’ Theorem in Ophthalmodynamometry

By E. F. Vastola, M.D., and T. Kokubu, M.D.

Numerous investigators have established the importance of ophthalmodynamometry in the diagnosis by safe and simple means of occlusion of the carotid artery proximal to the origin of the ophthalmic artery. In all their reports the significant observations are summarized by a statement giving the percentage of patients with proved carotid occlusion that was found to have significant lowering of the retinal artery pressure (RAP) on the side of occlusion. In the language of mathematical probability this percentage is the conditional probability per cent of significant RAP lowering given the condition of carotid occlusion. The highest value of 100 per cent for this probability was found by wood and Toole, who studied five patients with proved carotid occlusion and the lowest value of 62 per cent by Croll et al., who studied 16 such patients. The value for this conditional probability is a measure of the degree of confidence with which one may interpret the finding of normal RAP measurements but it is not a direct answer to the question usually asked by the clinician, i.e., "What are the chances that this patient with a recent cerebral occlusive vascular lesion and normal RAP measurements has in fact a patent carotid artery?" In more precise terms, a value is required for the conditional probability that the carotid artery is patent in a patient with a recent cerebral occlusive vascular lesion given the condition of normal RAP. Bayes’ theorem, a simple derivation in elementary statistical mathematics, identifies this conditional probability with an expression composed of terms for which values can be found by direct observation and the required value so obtained by calculation. Bayes’ theorem is seldom used in the interpretation of clinical information, and it is believed that a demonstration of its application to the results of ophthalmodynamometry might be of some interest. The theorem can be stated as follows:

\[ P(A/B) = \frac{P(A) \cdot P(B/A)}{P(B)} \]  

where \( A \) is the occurrence of patent carotid arteries in a patient with a recent cerebral occlusive vascular lesion, \( B \) is the occurrence of normal RAP measurements in a patient with a recent cerebral occlusive vascular lesion, \( P(A/B) \) (read as the probability of \( A \), given \( B \)) is the conditional probability of patent carotid arteries in a patient with a recent cerebral occlusive vascular lesion given the condition of normal RAP, \( P(A) \) is the unconditional probability of patent carotid arteries in a patient with a recent cerebral occlusive vascular lesion given the condition of normal RAP, \( P(B/A) \) is the conditional probability of normal RAP in a patient with a recent cerebral occlusive vascular lesion given the condition of patent carotid arteries, and \( P(B) \) is the unconditional probability of normal RAP in a patient with a recent cerebral occlusive vascular lesion. From the observations summarized below will be derived the values to be assigned to the three terms on the right side of (1).

Method

Retinal artery pressures were measured with a Baillart ophthalmodynamometer in the usual manner as described by Spelter. The pressures were recorded in grams, as given by the dial reading of the Baillart instrument, and the per cent difference between the two eyes was calculated for both systolic and diastolic pressures as the absolute value of the differences in pressures divided by the higher pressure and multiplied by 100 (table 1). All of the patients with an intracranial lesion and most of the controls had complete neurologic examinations, skull and chest...
x-rays, routine blood chemistries, and lumbar puncture. Many of the patients with an intracranial lesion, in addition, had further diagnostic procedures such as electroencephalography, air studies, and carotid angiography.

1. Control Group

Results

The criteria for abnormality were based upon the results of RAP measurements in 32 controls without evidence of organic intracranial disease. The average per cent difference was found to be four for the systolic pressure and six for the diastolic pressure with standard deviations of three and six respectively (table 1). The upper limit for normal RAP differences was placed at 15 per cent for the systolic pressure and 20 per cent for the diastolic pressure, 3.7 S.D.'s from the average difference for the former and 2.3 S.D.'s from the average difference for the latter. Only differences equal to or greater than these values were considered abnormal in all of the observations reported below. The ultimate justification of this choice of values for the normal range rests upon the fact that these values were found to maximize the predictive value of RAP measurements in the group of 59 patients described in section 4.

2. Patient Group

Ophthalmodynamometry was performed on 375 patients with recent, unilateral, cerebral lesions. No consistent efforts were made to select these patients for any particular characteristic; they were in large part successive admissions to an acute neurologic service from whom reliable RAP measurements could be obtained. Of these, 177 were believed on the basis of all available diagnostic procedures to have occlusive vascular lesions.

3. Determination of P(B)

In the group of 177 patients with occlusive lesions, 37 had lowering of both systolic and diastolic pressures and seven had lowering of systolic or diastolic pressures alone. Lowering beyond the normal range of either the systolic or diastolic pressure alone is considered significant, since six instances of the former and four instances of the latter occurred in patients with proved carotid occlusion. This gives a total of 44/177 or 25 per cent as the unconditional probability of abnormal RAP measurements and (100 - 25) = 75 per cent as the unconditional probability of normal RAP = P(B).

4. Determination of P(A)

In the total group of patients there were 59 in whom the results of ophthalmodynamometry could be compared with the results of angiography (57) or direct visualization (2). Ophthalmodynamometry was always performed before angiography. Seventeen of these patients were found to have a thrombotic occlusion of the carotid artery proximal to the origin of the ophthalmic artery; 14 of these had depressed RAP values, and in three both systolic and diastolic differences were within the normal range. This value of 3/17 or 18 per cent for "false negatives" may be used to find the true incidence (x) of carotid occlusion in the group of 177 patients with occlusive cerebral vascular lesions by solving the following:

\[
\frac{x - 44}{x} = 18
\]

(2)

The unconditional probability, therefore, of carotid occlusion in a patient with a recent, occlusive cerebral vascular lesion may be calculated to be 54/177 or 30 per cent and the unconditional probability of patent carotid arteries in such a patient is (100 - 30) or 70
per cent = P(A). In the studies reviewed by Silverstein\textsuperscript{14} carotid occlusion was found by angiography or postmortem examination in 14 to 39 per cent of patients with proved or suspected arteriosclerotic cerebrovascular disease.

5. Determination of P(B/A)

Of the 42 patients shown by angiography to have patent carotid arteries, none had significant lowering of either systolic or diastolic RAP. In agreement with most authors, therefore, we conclude that the conditional probability in patients with recent cerebral occlusive vascular lesions of normal RAP measurements given the condition that the carotid arteries are patent is 100 per cent = P(B/A).

6. Determination of P(A/B)

Substitution into (1) of the values for P(B), P(A) and P(B/A) found in sections 3, 4, and 5 gives:

$$P(A/B) = \frac{70}{75} = .93 \text{ or } 93\% \quad (3)$$

The critical determination for the final calculation in (3) concerns the incidence of significantly lowered RAP in a group of patients with recent cerebral occlusive vascular lesions. It may be objected that our finding of 25 per cent is too high. If 15 per cent be accepted as more realistic, P(B) becomes 85 per cent, P(A) becomes 82 per cent, and P(A/B) = 97 per cent.

Discussion

The value of 3 per cent which we have found for the conditional probability of patent carotid arteries in patients with recent cerebral occlusive vascular lesions given the finding of normal RAP, \(P(A/B)\), indicates that ophthalmodynamometry has greater diagnostic value than would have been inferred from our finding that 18 per cent of patients with proved carotid thrombotic occlusion had normal RAP. The 7 per cent error, however, cannot be disregarded. Normal RAP values in this group of "false negatives" probably represent restoration of normal blood supply by collateral vessels. At present we perform angiography in search of a resectable lesion of the extracranial portion of the carotid artery in a patient with normal RAP only if some particular feature of the case outweighs the results of ophthalmodynamometry.

Conclusions

1. The average per cent difference in retinal artery pressures in 32 normal controls was 4 per cent for the systolic and 6 per cent for the diastolic pressures; the corresponding standard deviations were 3 per cent and 6 per cent respectively.

2. The upper limit of the per cent differences that minimized the number of both false positives and false negatives in a series of 59 patients with angiographic or direct visualization of the carotid arteries was found to be 15 per cent for the systolic and 20 per cent for the diastolic pressures. The incidence of false positives in this series was negligible. The incidence of "false negatives" in those patients with thrombotic occlusion was 18 per cent.

3. The true incidence of carotid occlusion in our patients with recent cerebral occlusive vascular lesions was calculated from our findings to be 30 per cent.

4. The probability that the carotid arteries are patent in a patient with a recent cerebral occlusive vascular lesion when ophthalmodynamometry reveals normal RAP was calculated to be 93 per cent.

References


Bayes' Theorem


Giovanni Battista Morgagni, the Founder of Pathologic Anatomy

Giovanni Battista Morgagni was born on February 25, 1682. His birthplace was Forli, the capital of a papal province in northern Italy called Romagna, from the ancient Romandiola, the military army of the Roman legions.

After attending l'Accademia de'Filgergiti, a preparatory school in his native town, Morgagni enrolled, at fifteen years of age, in the Medical School of the University of Bologna, which at that time was in its waning glory. After four years of dedicated study at the University of Bologna, Morgagni received both the degree of Doctor of Medicine and that of Doctor of Philosophy.

After graduation, not yet twenty-one years of age, Morgagni was asked to assume Valsalva's teaching obligations at the University of Bologna while Valsalva was in Parma. Morgagni's infinite enthusiasm for his work made him a leader and a moving spirit among the medical students and young graduates. Thus, he was interested in and became president of the Academia inquietorum, which, literally translated, means "Academy of the Restless." The principle governing the Academy was that scientific truth should be based on analytic observation rather than on empirical theorizing and quotations from the classics.

It was to the "restless" that Morgagni presented his first original essays, soon to be published under the title, Adversaria anatomica (1706)—adversaria being the Latin name for notebooks. This marked the beginning of his scholarly career. The first collection of anatomical observations was followed by five more, with the last one coming thirteen years later.—C. G. Tedeschi, M.D. Giovanni Battista Morgagni, The Founder of Pathologic Anatomy: A Biographic Sketch On the Occasion of the 200th Anniversary Of the Publication Of His "De sedibus et causis morborum per anatomen indagatis." The Boston Medical Quarterly 12: 113, 1961.
Use of Bayes' Theorem in Ophthalmodynamometry
E. F. VASTOLA and T. KOKUBU

Circulation. 1962;26:1312-1315
doi: 10.1161/01.CIR.26.6.1312

The online version of this article, along with updated information and services, is located on the World Wide Web at:
http://circ.ahajournals.org/content/26/6/1312.citation

Permissions: Requests for permissions to reproduce figures, tables, or portions of articles originally published in Circulation can be obtained via RightsLink, a service of the Copyright Clearance Center, not the Editorial Office. Once the online version of the published article for which permission is being requested is located, click Request Permissions in the middle column of the Web page under Services. Further information about this process is available in the Permissions and Rights Question and Answer document.

Reprints: Information about reprints can be found online at:
http://www.lww.com/reprints

Subscriptions: Information about subscribing to Circulation is online at:
http://circ.ahajournals.org//subscriptions/