Comparison of Heart Vectors Calculated with Different Systems of Leads

By Richard McFee, M. S.

Einthoven's equations for the heart vector are based on the assumption that voltages measured across the trunk horizontally (or vertically) are proportional to the horizontal (or vertical) component of the dipole moment of the heart's electromotive forces. One necessary consequence of this assumption is that all the horizontal (or vertical) voltages be similar, and that heart vectors calculated with different sets of these voltages and appropriate formulas be the same. An experimental test of this predicted result is described in this paper.

INTRODUCTION

The experimental data, formulas, and procedures discussed in this paper represent an effort to measure the errors involved in the use of the equations based on the Einthoven triangle. It was hoped that by comparing vectors calculated from several different systems of leads taken on normal human subjects a reliable estimate of the errors involved in the application of these equations and a rational basis for judging arguments about their validity would be obtained. The investigation of the natural field of the heart eliminates many objections applicable to experiments involving artificial electromotive forces, models and mathematical approximations. The original idea was that the errors resulting from the use of the Einthoven formulas would be of about the same magnitude as the differences between the heart vectors calculated from the different systems of leads, but it was eventually realized that this view was not sound, for there could be perfect agreement among the vectors of the different systems of leads even with pronounced variations in the conductivity of the tissues in and around the heart. Moreover, there would be no differences between those vectors if their vertical and horizontal components found with the different systems of leads were equally in error. For these reasons it was not possible to equate the differences among the vectors to the errors of the formulas, although it seems probable that these differences are a rough index of the accuracy of the latter.

The comparison of these heart vectors is discussed in a later section of this article. The appendix gives equations for calculating the errors in the vectors resulting from inaccurate measuring of the lead voltages. These equations were developed for the purpose of estimating to what extent the differences between corresponding vectors are due to errors in measurement and to what extent they are due to errors inherent in the method of calculating these vectors. They can be applied to any calculation of the heart vector based on Einthoven's triangle and have been found useful in finding the errors in measurements of the ventricular gradient.

METHOD

Three different systems of leads were used. The first was the usual Einthoven system. The second consisted of an inverted Einthoven triangle formed by two electrodes on the left and right sides of the waist halfway between the ribs and the pelvic bone, and a third electrode on the right side of the neck. The third system included a horizontal lead taken from electrodes on opposite sides of the chest about two-and-one-half inches below the armpits, and a vertical lead taken from between the neck electrode to an electrode on the abdomen about an inch below the navel. These lead systems are shown in figure 1.

All the voltages of each lead system were recorded simultaneously on one film, using a three-channel.
vacuum tube electrocardiograph. The film speed was 450 mm. per second, and the sensitivity was about 2 cm. per millivolt. A transparent and finely divided precision ruler was used to measure the vertical distance of the trace of the QRS deflections from points on the baseline separated by intervals of five milliseconds. The time measured voltages were found by multiplying each measurement by the calibration factor for the corresponding channel. These voltages were "adjusted" to their most probable values by using the formulas given in the appendix.

The usual Einthoven equations,

\[ \alpha = \arctan \frac{2e_2 - e_1}{e_1 \sqrt{3}} \quad M = \frac{e_1}{\cos \alpha} \]  

were used for the two triangle systems, and the equations

\[ \alpha = \arctan \frac{e_x}{\frac{1}{2} e_H} \quad M = \sqrt{(e_x)^2 + (\frac{1}{2} e_H)^2} \]  

were used for the vertical-horizontal lead system. The horizontal voltage was arbitrarily divided by two, because the chest electrodes were nearer the heart than the arm and waist electrodes, for it was noted that the voltage across the chest was generally about twice as large as the corresponding voltages of the triangle systems; that is, of Lead I and the waist lead. The probable errors of the computed direction and the manifest magnitude of the cardiac vector due to mistakes in measurement were calculated by using the formulas given in the appendix. The time in the cardiac cycle of each measured voltage, with respect to an instant slightly before the onset of the complex, was determined. It is possible that errors as great as three or four milliseconds, one way or the other, were made in doing this.

**Results**

Graphs showing the data collected in this way are reproduced in figure 2. These superimposed curves give the manifest magnitude of the heart vector and the angle which defines the direction for each subject plotted against time. It will be noted that on each graph two curves are drawn for the ordinary Einthoven system of leads. These two curves were obtained from different heart beats and show the variations that result from using different beats for the calculations under consideration. It can be seen from the graphs that in each case these variations are relatively small.

These graphs show in detail the information obtained from the experiments. Of particular interest are the differences in angle and in magnitude between the highest and lowest curves of the graphs. As was mentioned in the introduction, these differences may be thought of as a rough index of the accuracy of the Einthoven formulas. For the convenience of the reader, average and maximum values for the differences of angle, along with average values for their probable errors and variations between beats are given in table 1. Corresponding differences in the magnitude of the vectors were not tabulated, because it was felt that the small errors made in locating the time scales would make the resulting differences misleadingly large.

In obtaining these numbers, attention was restricted to that portion of the QRS complex where the manifest magnitude exceeds one quarter its maximum value. This was done to minimize the large errors in measurement which are always associated with small deflections. The differences between the highest
and lowest curves were determined at instants five milliseconds apart in this interval, and the maximum and average of the results were tabulated. The differences between the two curves calculated with the Einthoven lead system were determined at the same instants, and also averaged and tabulated. The probable error figure was obtained by averaging the calculable probable error of all measurements in the interval considered.

**FIG. 2.** Time graphs of QRS electrical axis and magnitude calculated from different lead systems for three subjects. To indicate variations from beat to beat, calculations were made with the Einthoven triangle lead system for two different complexes.

**DISCUSSION**

There is no doubt that the inverted Einthoven triangle and the vertical-horizontal lead systems used here are completely arbitrary, as
are the equations which accompany them. But the Einthoven triangle itself was chosen on an arbitrary basis, and there seems to be no basic reason why any one of the lead systems should be considered superior to any of the others. The equations for the leads may be chosen

Table 1.—Tabulated Values for the Differences Between the Curves in Figure 2.*

<table>
<thead>
<tr>
<th>Subject</th>
<th>Maximum Difference Between Highest and Lowest Curve</th>
<th>Average Difference Between Highest and Lowest Curve</th>
<th>Average Difference Between Curves of Same Lead System</th>
<th>Average Probable Error</th>
<th>Average Difference Between Curves of Same Lead System with Different Beats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject 1</td>
<td>14.9°</td>
<td>9.9°</td>
<td>1.9°</td>
<td>2.4°</td>
<td></td>
</tr>
<tr>
<td>Subject 2</td>
<td>69.8°</td>
<td>25.8°</td>
<td>1.9°</td>
<td>9.0°</td>
<td></td>
</tr>
<tr>
<td>Subject 3</td>
<td>58.8°</td>
<td>30.8°</td>
<td>1.9°</td>
<td>5.4°</td>
<td></td>
</tr>
</tbody>
</table>

* For detailed explanation, see text.

more rationally using the concept of a lead vector recently advanced by Burger and Van Milaan and finding the components of this vector with a suitable model, as they did. Or they may be chosen using a human subject and artificially inducing a dipole, as was done by Wilson and his associates. But in any case, by comparing heart vectors calculated from different systems of leads with formulas obtained by such methods, we can test their validity when applied to human subjects, and in this manner find out which ways of calculating the heart vector are best.

The procedure of comparing vectors calculated with different lead systems suggests a method of constructing a lead which is more uniformly sensitive to electromotive forces generated in different parts of the heart. This is done by using two or more electrodes for each end of the lead and averaging their potentials. The electrodes are located so that the electromotive forces they emphasize are in opposite portions of the heart. This emphasis, as a result, will be cancelled out when their potentials are averaged. Actual averaging networks are constructed by connecting the electrodes with equal resistances to the same point, as is done with the central terminal. It is easily shown that the potential of this point will be the average. The requirement that the sum of the currents flowing into the point be zero (Kirchhoff's current law) is equivalent to the equation

\[
\frac{V_1 - V_a}{R} + \frac{V_2 - V_a}{R} + \ldots + \frac{V_n - V_a}{R} = 0
\]

(3)

\[
n = 2, 3, 4, \ldots
\]
where \( V_1, V_2, \ldots, V_n \) are the potentials of the \( n \) electrodes and \( V_a \) is the potential of the output terminal. Solving this equation for \( V_a \) gives

\[
V_a = \frac{1}{N} (V_1 + V_2 + \cdots + V_n)
\]

which is the desired average.

Some examples of these averaging systems are shown in figure 3. Part A of this figure shows an application to the tetrahedron of Wilson, Johnston and Kossmann. The central point of the averaging network replaces the usual sagittal electrode of the tetrahedron. In this way a sagittal electrode is obtained which is effectively more “remote” from the heart. Part B shows a symmetrical system for obtaining all three components of the heart vector with eight electrodes. Each component is the average of four bipolar leads. If the output voltages of this latter system are considered directly proportional to the three components of the heart vector, proper weighting factors for each component may be introduced by “adjusting” the gain of the amplifiers. If Burger’s equations for the output leads are used, simple resistive networks may be constructed for obtaining linear combinations of the output voltages, and they can be adjusted so that the combined voltages are proportional to the desired components.

### SUMMARY

The instantaneous magnitude and direction of the cardiac vector is calculated for the QRS complex of three subjects using the Einthoven lead system, an inverted Einthoven lead system and a two lead “vertical-horizontal” lead system. Comparison of the time graphs of the results gives a qualitative idea of the accuracy of the Einthoven formulas.

Equations are given for estimating the probable errors in the manifest vector resulting from inaccurate measurement of the voltages of the leads.

### APPENDIX

*Equations for estimating the errors resulting from inaccurate measurement of the voltages of the leads:*

These formulas may be used only for calculations based on the ordinary Einthoven triangle.

**Equations for estimating the errors resulting from inaccurate measurement of the voltages of the leads:**

\[
V_a = \frac{1}{N} (V_1 + V_2 + \cdots + V_n)
\]

where

\[
\Delta = I + III - II
\]

**Probable error in angle \((\alpha) = \pm 40 \frac{\Delta}{M} \text{ degrees}\quad (1)\)**

**Probable error in manifest magnitude \((M) = \pm 80 \frac{\Delta}{M} \text{ per cent}\quad (2)\)**

The probable error of a measurement is that quantity which will, on the average, exceed and be exceeded by half of the actual errors.

The influence of measurement errors may be reduced about forty percent by “adjusting” the lead voltages so that they conform with Einthoven’s Law. The equations for the adjusted values are

\[
\begin{align*}
I_a &= I - \frac{1}{2} \Delta \\
II_a &= II + \frac{1}{2} \Delta \\
III_a &= III - \frac{1}{2} \Delta
\end{align*}
\]

When these adjusted lead voltages are used the formulas for the measurement of errors become

**Probable error in \(\alpha = \pm 25 \frac{\Delta}{M} \text{ degrees}\quad (5)\)**

**Probable error in \(M = \pm 50 \frac{\Delta}{M} \text{ per cent}\quad (6)\)**

The equations for the probable error of the measurements are likely to be in error by as much as 100 per cent, but they are constructed so that on the average half the results obtained with them will be larger than the true probable errors, and half will be smaller.

The derivations of these formulas are fairly lengthy and will not be given here. The general idea is as follows. It is assumed that there is an equal distribution of random errors in the measured values of all lead voltages. The probable error of the \(\Delta\) distribution, \(\epsilon\), can then be shown to be related to the probable errors in the individual leads, \(\epsilon_l\), by

\[
\epsilon_a = \sqrt{3} \epsilon_l
\]

The probable errors in the individual leads are found by letting

\[
\epsilon_a = \Delta
\]
This assumption (8) leads to results which will be too large half the time and too small the other half. The probable errors for \( \alpha \) and \( M \) are then found using Einthoven’s equations and the conventional formula for the propagation of errors.

\[
\text{Probable error of } F(x, y, z) = \sqrt{\left(\frac{\partial F}{\partial x} \epsilon_x\right)^2 + \left(\frac{\partial F}{\partial y} \epsilon_y\right)^2 + \left(\frac{\partial F}{\partial z} \epsilon_z\right)^2} \quad (9)
\]

where \( \epsilon_x \), \( \epsilon_y \), and \( \epsilon_z \) are the probable errors of \( x \), \( y \), and \( z \).

When adjusted values for the lead voltages are used, errors of each lead must be identified and added linearly, instead of in a root mean square fashion as is done for errors from different leads.

The adjusted values for the lead voltages given in equation (4) are such that they simultaneously satisfy Einthoven’s law and the requirement that the root mean square of the difference between them and the measured values (“residuals”) be minimum. It can be shown that these are the most probable values of the lead voltages.

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**References**

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