Standardizing Factors in Electrocardiography

By James A. Cronvich, M.S. John P. Conway, M.D. and George E. Burch, M.D.

Modifications in recorder standardization are required to correct inherent scalar differences among potential differences measured in a given electrocardiographic reference frame. In the frontal plane, agreement between the resultant vectors calculated from unipolar limb lead measurements and from standard lead measurements can be obtained by standardizing the galvanometer for recording the unipolar limb lead potentials at $\sqrt{3}$ cm. per millivolt. A method for calculating this "standardizing factor" and similar factors for other reference frames is presented and applied to calculations for the equilateral tetrahedron, the isosceles tetrahedron, and the cube.

INTRODUCTION

RECENT interest in spatial electrocardiography and vectorcardiography has resulted in the introduction of several spatial coordinate reference frames. A convenient method for correlating the data obtained in any such frame seems desirable. Need for such correlation in the frontal plane was indicated with the introduction of unipolar limb leads when corresponding vectors determined from measurements with the unipolar limb leads and with the standard leads were found to differ in magnitude. To secure agreement, measurements from the unipolar limb leads had to be multiplied by the factor $\sqrt{3}$ or the galvanometer had to be standardized at $\sqrt{3}$ cm. per millivolt for recording unipolar limb lead potentials. As proved by Hill, this "standardizing factor" $\sqrt{3}$ is required because the potential differences measured on the galvanometer are scalar quantities which are treated as vectors when added in the triaxial reference system. This difference between the sets of measurements is considered in Wilson's method of vectorcardiography in the frontal plane when the sensitivity of the central terminal-to-foot circuit (a unipolar limb lead) is made $\sqrt{3}$ times that of the Lead I circuit. It is apparent that standardizing factors are required in spatial electrocardiography and vectorcardiography also.

In this paper two theorems are presented which facilitate calculation of standardizing factors for certain two- and three-dimensional frames whose boundary points represent electrode positions assumed to be equidistant electrically from the dipole of the heart. Application of these theorems to the calculation of standardizing factors for the Einthoven frontal plane and the equilateral tetrahedron is presented in detail.

THEOREMS

I. The potential difference between two remote points coplanar with a dipole and equidistant from its center is proportional to the projected distance between these points on the axis of the dipole.

Proof: Consider points A and B with coordinates $x_1$, $y_1$, and $x_2$, $y_2$, respectively, both at distance $r$ from the center of the dipole. It is assumed that the dipole of moment $D$ lies along the z axis with its center at 0 and that $r$ is much greater than the distance between the dipole charges (fig. 1).

The following equations apply for the potentials produced by the dipole at points A and B:

$$E_A = \frac{KD}{r^2} \cos \theta_1 = \frac{KD}{r^2} \cdot \frac{x_1}{r} = \frac{KD}{r^3} \cdot x_1$$

$$E_B = \frac{KD}{r^2} \cos \theta_2 = \frac{KD}{r^2} \cdot \frac{x_2}{r} = \frac{KD}{r^3} \cdot x_2$$

where $K$ is a proportionality constant. Therefore,

$$E_A - E_B = \frac{KD}{r^3} (x_1 - x_2)$$

where $(x_1 - x_2)$ is the projected distance between the two points on the axis of the dipole.

Discussion: If we consider two pairs of points $L(x_3, y_3)$ and $M(x_4, y_4)$, and $R(x_5, y_5)$ and $S(x_6, y_6)$...
STANDARDIZING FACTORS IN ELECTROCARDIOGRAPHY

all at distance \( r \) from the center of the dipole with \( y_1 = y_2 = y \), then \( L \) and \( M \) lie on one line parallel to the dipole and \( R \) and \( S \) lie on another line parallel to the dipole.

By application of theorem I,

\[
\begin{align*}
E_L - E_M &= E_{LM} = \frac{KD}{r^3} (x_3 - x_1) = \frac{KD}{r^3} \cdot \overline{LM} \\
E_R - E_S &= E_{RS} = \frac{KD}{r^3} (x_5 - x_6) = \frac{KD}{r^3} \cdot \overline{RS}
\end{align*}
\]

(3) \hspace{2cm} (4)

where \( \overline{LM} \) and \( \overline{RS} \) are the distances between the respective points. Dividing equation 3 by equation 4, we obtain

\[
\frac{E_{LM}}{E_{RS}} = \frac{\overline{LM}}{\overline{RS}}
\]

(5)

In this case the same dipole produced potential differences \( E_{LM} \) and \( E_{RS} \) between two pairs of equally remote points. However, if \( \overline{LM} \) is greater than \( \overline{RS} \), then \( E_{LM} \) is greater than \( E_{RS} \) by the ratio \( \overline{LM}/\overline{RS} \). If \( E_{LM} \) is taken as the reference potential difference to determine the dipole moment, potential difference \( E_{RS} \) must be multiplied by the factor \( \overline{LM}/\overline{RS} \) to make the measurements of potential difference alone indicate equal dipole moments. Consequently \( \overline{LM}/\overline{RS} \) is the standardizing factor which must be applied to \( E_{RS} \). Therefore, the standardizing factor for potential differences between pairs of points equally remote from a dipole is inversely proportional to the distances between the points.

(Note: For simplicity the preceding analysis was made in electrostatics. However, if the conducting medium to which the analysis is applied is electrically homogeneous and isotropic, the same ratios will apply to potential differences with current flowing.)

II. The potential at the junction of \( n \) equal impedances connected to \( n \) points in space is the algebraic average of the potential of the \( n \) points.

Proof: This is readily shown by application of an electrical network theorem of Millman. Consider the circuit in figure 2.

The points \( I, 2, 3, 4, \ldots N \) are at potentials \( E_1, E_2, E_3, E_4, \ldots E_N \), respectively, with respect to the reference point 0. By Millman's theorem, the potential of point \( C \), the junction of the \( n \) impedances, is

\[
E_c = \frac{\sum_{s=1}^{n} \frac{E_s}{Z_s} \cdot \frac{1}{Z}}{\sum_{s=1}^{n} \frac{1}{Z_s}} = \frac{1}{Z} \left( \frac{E_1 + E_2 + E_3 + \cdots + E_N}{Z} \right)
\]

(6)

If \( Z_1 = Z_2 = Z_3 = \cdots Z_N = Z \)

then

\[
E_c = \frac{1}{Z} \left( \frac{E_1 + E_2 + E_3 + \cdots + E_N}{Z} \right)
\]

(7)

Therefore, \( E_c \) is the algebraic mean of the \( n \) potentials.

FIG. 1. Dipole at \( O \) and several points equidistant from \( O \) in plane \( XY \).
Discussion: This theorem is useful for those cases in which potential differences are measured between the Wilson central terminal or similar balanced central terminals and an exploring electrode. For the particular reference frame and central terminal configuration selected, it facilitates the calculation of the potential of the central terminal. However, if the impedances between the central terminal and the electrode positions on the body are unequal, the potential of the terminal is calculated by use of equation 6.

As a corollary of this theorem it should be noted that potential \( E_c \) cannot be greater than the maximum potential of points \( 1, 2, 3, 4, \ldots N \).

Applications

In the applications considered in this paper the following are assumed:

1. The center of the dipole is electrically equidistant from the electrodes between which the potential differences are measured.

2. The medium in which the dipole is immersed is electrically homogeneous and isotropic and infinite in extent. These assumptions are common to the Einthoven concept of electrocardiography in the frontal plane and are here applied to spatial vectorcardiography also.

The mathematical procedure is simplified if the calculations for standardizing factor are based not on the ratio of the lengths of the vectors resulting from the different sets of measurements for a fixed dipole, but rather on the maximal potential differences which may be obtained in any one set if a given dipole is rotated in all directions about its center. The sets are chosen so that all maximal possible readings in the set are equal. From theorem I these maxima are obtained when the dipole is parallel to the line connecting the electrode positions in the coordinate frame.

I. FRONTAL PLANE (Einthoven's equilateral triangle with Wilson's central terminal. Dipole at center of triangle. Figure 3)

All resistors \( R_1 \) are equal. Therefore, from theorem I

\[
e_c = \frac{e_R + e_L + e_F}{3}.
\]

Since the dipole lies in the frontal plane with its center coinciding with that of the triangle,

\[
e_R + e_L + e_F = 0.
\]

Therefore,

\[
e_c = 0.
\]

In this case, the potential of the central terminal is zero. From the symmetric disposition of the equal positive and negative dipole charges about point \( O \), the potential at point \( O \) is zero. The central terminal and the center of the dipole may be said to coincide electrically. If the dipole is parallel to \( RL \), the potential difference between \( R \) and \( L \), \( e_1 \), will be maximal, and by theorem I

\[
e_{l\text{max}} = \frac{KD}{r^2} \cdot RL
\]

Similarly with the dipole parallel to \( FR \),

\[
e_{f\text{max}} = \frac{KD}{r^2} \cdot FR
\]

and with the dipole parallel to \( LF \),

\[
e_{l\text{max}} = \frac{KD}{r^2} \cdot LF
\]

Since the sides of an equilateral triangle are equal,

\[
RL = LF = FR
\]

and

\[
e_{l\text{max}} = e_{f\text{max}} = e_{l\text{max}}.
\]
STANDARDIZING FACTORS IN ELECTROCARDIOGRAPHY

Therefore, if a dipole located at the center of the frontal plane is rotated in this plane, about its center, it will produce equal but, of course, not simultaneous maximal potential differences in each of the standard leads.

As shown, O and C are both at zero potential. Therefore, by theorem I, if the dipole is directed along OR

\[ e_{OR_{\text{max}}} = e_{CR_{\text{max}}} = e_{FR_{\text{max}}} = \frac{KD}{r^3} \cdot OR \] (16)

Similarly, if the dipole is directed along OL or along OF

\[ e_{OL_{\text{max}}} = \frac{KD}{r^3} \cdot OL \quad \text{or} \quad e_{OF_{\text{max}}} = \frac{KD}{r^3} \cdot OF \] (17)

The relations among the medians and the sides of an equilateral triangle are

\[ OR = \sqrt{3} \cdot OI = \sqrt{3} = OF = RF = LF = FR \] (18)

Therefore,

\[ \sqrt{\frac{3}{3}} = e_{OL_{\text{max}}} = e_{OF_{\text{max}}} = \sqrt{\frac{3}{3}} \]

Consequently, although the maximal potential differences produced by a dipole as it rotates about the center of the frontal plane triangle are equal in the standard leads or in the unipolar limb leads, the maximal potential difference in the standard leads is greater by \( \sqrt{3} \) than that of the limb leads. If the standard leads are taken as reference, then the standardizing factor to be applied to the unipolar lead measurements is \( \sqrt{3} = 1.732 \), which is the ratio of the length of the side to the median of the equilateral triangle.

II. EQUILATERAL TETRAHEDRON

**Case I.** (Central terminal formed by joining equal resistors \( R_1 \) to the four apices of the tetrahedron. Dipole at center of tetrahedron. Fig. 4a).—From theorem I, the maximal potential differences between pairs of apices \( R, L, F, \) and \( B \) are

\[ e_{RL_{\text{max}}} = \frac{KD}{r^3} \cdot RL; \quad e_{LF_{\text{max}}} = \frac{KD}{r^3} \cdot LF; \quad \text{etc.} \] (19)

But all edges of the equilateral tetrahedron are equal, that is

\[ RL = LF = FR = FB = iB = FB \] (20)

\[ R = LF = FR = FB \] (21)

as the dipole is rotated in all directions about the center of the tetrahedron.

Similarly, the maximal potentials at the apices are

\[ e_{R_{\text{max}}} = \frac{KD}{r^3} \cdot OR; \quad e_{L_{\text{max}}} = \frac{KD}{r^3} \cdot OL; \]

\[ e_{F_{\text{max}}} = \frac{KD}{r^3} \cdot OF \quad \text{and} \quad e_{B_{\text{max}}} = \frac{KD}{r^3} \cdot OB \] (22)

But all medians of the equilateral tetrahedron are equal, or

\[ OR = OL = OF = OB = r \] (23)

Therefore,

\[ e_{R_{\text{max}}} = e_{L_{\text{max}}} = e_{F_{\text{max}}} = e_{B_{\text{max}}} \] (24)
Since points $R$, $L$, $F$, and $B$ are symmetrically disposed with respect to the center of the dipole,

$$e_R + e_L + e_F + e_B = 0$$

for any direction of the dipole. Moreover, the resistors connecting $C$ to $R$, $L$, $F$, and $B$ are equal. Consequently, by theorem II,

$$e_C = 0$$

The central terminal in this case is at zero potential for any direction of the dipole and may be said to coincide electrically with the center of the dipole. Hence

$$e_{CR} = e_R; \quad e_{CL} = e_L; \quad e_{CF} = e_F; \quad e_{CB} = e_B.$$  \hspace{1cm} (27)

As shown in appendix I,

$$\overline{OR} \left(2 \sqrt{\frac{2}{3}}\right) = RL, \ldots, \text{etc.}$$  \hspace{1cm} (28)

Therefore,

$$e_{CR\text{max}} \left(2 \sqrt{\frac{2}{3}}\right) = e_{CL\text{max}} \left(2 \sqrt{\frac{2}{3}}\right) = e_{CF\text{max}} \left(2 \sqrt{\frac{2}{3}}\right) = e_{CB\text{max}} \left(2 \sqrt{\frac{2}{3}}\right)$$

$$= e_{RL\text{max}} = e_{LF\text{max}} = e_{FR\text{max}}$$

$$= e_{RB\text{max}} = e_{LB\text{max}} = e_{FB\text{max}}.$$  \hspace{1cm} (29)

Consequently, if the measurements of potential differences between the apices of the tetrahedron, corresponding to four electrode positions, are taken as reference, the standardizing factor to be applied to the unipolar lead measurements is $2 \sqrt{\frac{2}{3}} = 1.627$.

Case II. (Wilson's central terminal in the frontal plane. Dipole at the center of the tetrahedron. Fig. 4b).—The same considerations as in case I apply to measurements between apices (between limb electrodes or between limb and back electrodes), but the standardizing factors for measurements from the frontal plane central terminal to the limbs and back must be determined.

The maximal potential difference between the central terminal and the back is obtained when the dipole lies along the median $\overline{OR}$ to the back. In this position the dipole is symmetrically located with respect to points $R$, $L$, and $F$, that is, the distances between these points and the center of the dipole are equal, and the angles between the medians connecting these points to the dipole and the axis of the dipole are equal. Therefore,

$$e_R = e_L = e_F.$$  \hspace{1cm} (30)

Since terminal $C$ is connected to $R$, $L$, and $F$ by three equal resistors, $R_1$,

$$e_C = \frac{e_R + e_L + e_F}{3} = e_B.$$  \hspace{1cm} (31)

With the dipole in this position, according to theorem I,

$$e_{RB\text{max}} = \frac{KD}{r^2} \cdot \overline{AB}.$$  \hspace{1cm} (32)

where $A$ is the midpoint of the frontal plane and $\overline{AB}$ is the projection of $\overline{RB}$ on the axis of the dipole. But $C$ and $R$ are at the same potential; therefore,

$$e_{CB\text{max}} = \frac{KD}{r^2} \cdot \overline{AR}.$$  \hspace{1cm} (33)

Maximal potential difference between $R$ and $B$ is obtained when the dipole is parallel to $\overline{RB}$. As shown in appendix I,

$$\overline{AB} \sqrt{\frac{3}{2}} = \overline{RB}.$$  \hspace{1cm} (34)

Therefore,

$$e_{CB\text{max}} \sqrt{\frac{3}{2}} = e_{RB\text{max}} = e_{RL\text{max}} = e_{LF\text{max}} = e_{FR\text{max}}.$$  \hspace{1cm} (35)

e etc.

and the standardizing factor to be applied to the central terminal-to-back measurements is

$$\sqrt{\frac{3}{2}} = 1.225.$$  \hspace{1cm} (36)

Now consider the potential differences between the central terminal and $R$, $L$, or $F$ (the limb electrodes). The maximal potential difference between the central terminal and $R$ will be obtained when the dipole is parallel to median $\overline{AR}$ in the frontal plane. Under this condition, by theorem I,

$$e_R = \frac{KD}{r^2} \cdot \overline{AR},$$  \hspace{1cm} (36)

where $\overline{AR}$ is the projection of $\overline{OR}$ onto the axis of the dipole.
Moreover, with the dipole parallel to the frontal plane,
\[ e_R + e_L + e_F = 0 \]  
(37)
and by theorem II,
\[ e_C = 0. \]  
(38)

Therefore,
\[ ec_{R\text{max}} = \frac{KD}{r^3} AR. \]  
(39)
Similarly
\[ ec_{L\text{max}} = \frac{KD}{r^3} AL \quad \text{and} \quad ec_{F\text{max}} = \frac{KD}{r^3} AF. \]  
(40)

Since $AR$, $AL$, and $AF$ are medians of the frontal plane triangle,
\[ \overline{AR} \sqrt{3} = AL \sqrt{3} = AF \sqrt{3} \]
\[ = RL = LF = FR = RB = LB = FB \]

Therefore,
\[ ec_{R\text{max}} \sqrt{3} = ec_{L\text{max}} \sqrt{3} = ec_{F\text{max}} \sqrt{3} \]
\[ = ec_{R\text{max}} \sqrt{3} = ec_{L\text{max}} \sqrt{3} = ec_{F\text{max}} \sqrt{3}, \text{etc.} \]  
(42)

and the standardizing factor to be applied to the central terminal-to-limb measurements is

\[ \sqrt{3} = 1.732, \text{ as in Einthoven frontal plane measurements where the dipole is assumed to lie in the frontal plane.} \]

The isosceles tetrahedron* and the cube shown in figures 5 and 6 respectively have been analyzed similarly. For reference the standardizing factors for all frames are tabulated in table 1.

Table 1.—Standardizing Factors for Potential Differences in Several Reference Frames

<table>
<thead>
<tr>
<th>Einthoven Triangle</th>
<th>Equilateral Tetrahedron Case I, Fig. 4a</th>
<th>Equilateral Tetrahedron Case II, Fig. 4b</th>
<th>Isosceles Tetrahedron Case I, Fig. 5</th>
<th>Cube Case II, Fig. 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{RL}$ = $e_1$</td>
<td>1</td>
<td>$e_{RL}$</td>
<td>1</td>
<td>$e_{RL}$</td>
</tr>
<tr>
<td>$e_{FR}$ = $e_2$</td>
<td>1</td>
<td>$e_{FR}$</td>
<td>1</td>
<td>$e_{FR}$</td>
</tr>
<tr>
<td>$e_{LR}$ = $e_3$</td>
<td>1</td>
<td>$e_{LR}$</td>
<td>1</td>
<td>$e_{LR}$</td>
</tr>
<tr>
<td>$e_{RL}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{RL}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{RL}$</td>
</tr>
<tr>
<td>$e_{FR}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{FR}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{FR}$</td>
</tr>
<tr>
<td>$e_{LR}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{LR}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{LR}$</td>
</tr>
<tr>
<td>$e_{RL}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{RL}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{RL}$</td>
</tr>
<tr>
<td>$e_{FR}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{FR}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{FR}$</td>
</tr>
<tr>
<td>$e_{LR}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{LR}$</td>
<td>$\sqrt{3}$</td>
<td>$e_{LR}$</td>
</tr>
</tbody>
</table>

* The name isosceles tetrahedron has been applied to one whose frontal plane (apices $R$, $L$, and $F$) is equilateral and whose frontal plane medians are equal to $RB$, $LB$, and $FB$. With the isosceles tetrahedron, it is assumed that the Wilson frontal plane central terminal is used and that the dipole lies at the center of this frontal plane equidistant from apices $R$, $L$, $F$, and $B$. With the cube, it is assumed that the central terminal is the junction of the equal resistors connecting the eight corners, and that the dipole lies at the center of the cube.

**Discussion**

Throughout this study the calculation of standardizing factors has been associated with the potential and the apparent location of the central terminal in the coordinate frame. For visualizing the apparent location we have found it convenient to think in terms of the following model: Each resistor in the electrical system is replaced by a rubber band. All bands have the same elasticity and cross section but their unstretched lengths are proportional to the resistances they replace. The bands are joined at one end to form the central terminal and the other ends are fastened to the correct apical points of the coordinate frame. As the bands stretch, the position of the junction corresponds to the apparent location of the central terminal.

This concept of the central terminal must be regarded with care, because the apparent location of the central terminal cannot be used generally to calculate the potential of the terminal. Consider, for example, the central terminal in case I of the analysis of the equi-
lateral tetrahedron. From our model the central terminal coincides with the center of the dipole at the center of the tetrahedron and its potential is zero, as is correct. However, if the dipole were directed along one of the medians and the resistor along that median were changed slightly, the position of the central terminal would be considered to move along that median and no longer to coincide with the dipole. If the potential of the central terminal were calculated by the inverse square law from the distance between the central terminal and the center of the dipole, a large but incorrect potential would result. As noted in the discussion of theorem I, the potential of the central terminal can be no higher than the highest potential of the apices. Consequently, the maximal possible potential of the central terminal would be \(KD/r^2\), which would occur only if the resistor were made zero so that the central terminal would be shifted to one of the apices. The potential of the central terminal is correctly determined in any case by application of Millman's theorem, equation 6. However, no error is made if the distances between this apparent position of the central terminal and the apices are used with theorem II to calculate the standardizing factors for the central terminal-to-apex potential differences.

A similar fallacy would arise in case II of the equilateral tetrahedron if the potential of the central terminal were determined from its apparent position at the center of the frontal plane and the inverse square law. No difficulty is encountered, however, if theorems I and II are employed.

If the center of the dipole and the apparent location of the central terminal do not coincide, the potential of the central terminal is not zero unless the dipole is perpendicular to the line connecting its center with the apparent location of the central terminal. A dipole located in front of or behind the frontal plane will, unless it is parallel to the frontal plane, produce a central terminal potential determined by the orientation of the dipole with respect to the apices of the frontal plane.

Application of the model and theorem I visually explains why Goldberger's augmented limb leads produce potential differences which are 1.5 times as great as those of Wilson's limb leads. In Goldberger's method the reference terminal is apparently shifted to the midpoint of one of the sides of the Einthoven triangle. Measurement of limb potential between this point and the limb involves a distance which is 1.5 times as large as that involved with Wilson's central terminal. This is shown in figure 7. The accuracy of Goldberger's method depends on equality of electrode-to-skin resistances. The central terminal is shifted.

---

**Fig. 5.** Isosceles tetrahedron with Wilson's central terminal in frontal plane.

**Fig. 6.** Reference system employing the cube.

**Fig. 7.** Comparison of Wilson's limb lead with Goldberger's augmented limb lead.
toward the electrode with the lower resistance. This shift can produce magnitude and polarity changes in some cases. The use of the 5000 ohm resistors in Wilson's method masks differences in electrode-to-skin resistances and reduces the magnitude of any error.

When the model for determining the apparent location of the central terminal is considered, the question might be asked: For determination of the limb potentials with a galvanometer with the use of Wilson's central terminal, does not the resistance of the galvanometer shunt one of the three equal resistors and thereby move the apparent location of the central terminal toward the limb being measured? The answer is: It does. Fortunately, however, the method of calibrating the galvanometer corrects any error. This is shown by applying Thevenin's theorem of electric circuits. Figure 8a is a schematic arrangement of the apparatus and the Einthoven triangle for determining the potential of the left arm by Wilson's method. Figure 8b is simply a rearrangement of figure 8a with the limb potentials \(e_K\), \(e_L\), and \(e_F\) made evident. If the galvanometer is disconnected, the network inside the dotted lines is open-circuited, and \(C\) and \(O\) are at the same potential. Therefore, the open-circuit output voltage, \(e_{CL}\), in this case is \(e_L\). The internal impedance of the network measured from terminals \(C\) and \(L\) into the box with \(e_K\), \(e_L\), and \(e_F\) short-circuited is \(\frac{R}{3}\). The equivalent Thevenin network of the circuit within the dotted lines is shown in figure 8c. Since both \(e_L\) and the calibrating voltage \(e\) are in series with \(R_1\) and \(\frac{R_1}{3}\), the method of calibrating the galvanometer compares \(e_L\) and the calibrating voltage directly. Therefore, the calibrated galvanometer indication is correctly \(e_L\).

The effect of skin-electrode resistances was mentioned in connection with the possibility of errors in the use of Goldberger's augmented

---

**Fig. 8.** Application of Thevenin's theorem of electrical networks to prove the accuracy of string galvanometer recordings of limb potentials. (a) Actual circuit arrangement. (b) Equivalent circuit arrangement. (c) Equivalent circuit arrangement according to Thevenin's theorem.
terminal-to-limb electrode potential differences. Thus, if the skin-electrode resistances are all 1000 ohms and the individual Wilson network resistors are 5000 ohms, the central terminal-to-limb potential differences as measured will be approximately 17 per cent less than they actually are, that is, the 1000 and 5000 ohms resistances are acting as a voltage divider to reduce the voltage presented to the galvanometer by $\frac{1000}{1000 + 5000} = \frac{1}{6} \approx 17\%$.

If the skin-electrode resistances are unequal, then a shift in the apparent position of the central terminal will occur which may introduce further errors. By careful application of the electrodes it is usually possible to reduce this error until it is within usual experimental electrocardiographic errors. However, in certain cases where the subject has very dry and thick skin, the error may be relatively large.

No attempt has been made in this paper to justify the validity of any coordinate frame for electrocardiography. Einthoven’s frontal plane concept has been the basis of two-dimensional electrocardiography. As yet there is no standardization of frames for spatial electrocardiography and vectorcardiography. At present one of the tetrahedrons seems most suitable but there is little difference between results obtained with the isosceles and the equilateral tetrahedron concepts. The errors inherent in the use of any coordinate frame for electrocardiography must be realized. The body is neither infinite in extent, nor electrically homogeneous and isotropic. The electrodes are not necessarily so far removed from the distributed charges of the active cardiac muscle that these charges can be represented accurately by a dipole. However, such errors are probably well within the range of variation due to normal physiologic differences, as the work of Wilson and others regarding the validity of the Einthoven concept seems to indicate.

**Summary**

1. Standardizing factors are necessary to correlate measurements in any given coordinate reference frame employed in electrocardiography.

2. Two theorems are presented which facilitate the calculation of standardizing factors.

3. The application of these theorems to the Einthoven triangle and to the equilateral tetrahedron for spatial electro- and vectorcardiography has been shown in detail.

4. Standardizing factors for the Einthoven triangle, the equilateral tetrahedron, the isosceles tetrahedron, and the cube have been tabulated.

5. A concept regarding the apparent location of the central terminal has been presented, and with the theorems, discussed with respect to some applications in electrocardiography.

**Addendum**

Just before this paper was submitted for publication, a similar paper, “Le Problème des Derivations Bipolaires Appliqué à la Vectocardio graphic,” by E. C. Cabrera appeared in Acta Cardiologica 4: 231, 1949. Whereas in our paper standardizing factors are based on the distance between the electrode locations projected onto the axis of the dipole, in Cabrera’s paper they are based on the sine of half the angle between the lines connecting the center of the dipole with the electrode positions. Mathematically, for equidistant electrodes the two methods are equivalent. In our paper, however, we consider particularly the tetrahedron systems which have been used in the vectorcardiographic investigations of this laboratory and similar systems in which derived electrodes (central terminals) as well as direct contact electrodes are used.

**Appendix I. Equilateral Tetrahedron**

Consider the sagittal section of the equilateral tetrahedron shown in figure 9. This section is obtained by passing a plane through B and F perpendicular to the frontal plane. Points O, the center of the tetrahedron, and A, the center of the frontal plane, lie in the plane of this section. In the equilateral tetrahedron

$$\overline{RL} = \overline{LF} = \overline{FR} = \overline{RB} = \overline{LB} = \overline{FB}$$
and
\[ \overline{OR} = \overline{OL} = \overline{OF} = \overline{OB} = r \]
In the section,
\[ \cos \alpha = \frac{\overline{FB}}{\overline{OB}} = \frac{\overline{AB}}{\overline{FB}} = \frac{\sqrt{\overline{FB}^2 - \overline{AF}^2}}{\overline{FB}} \]
Since \( \overline{AF} \) is the median of the equilateral frontal plane
\[ \overline{AF} = \frac{\overline{RL}}{\sqrt{3}} = \frac{\overline{FB}}{\sqrt{3}} \]

**Fig. 9.** Sagittal section of the equilateral tetrahedron.

Therefore,
\[ \frac{\frac{1}{2} \overline{FB} \sqrt{\overline{FB}^2 - \left( \frac{\overline{FB}}{\sqrt{3}} \right)^2}}{\overline{OB}} = \sqrt{\frac{2}{3}} \]
or
\[ \overline{OB} \left( 2 \sqrt{\frac{2}{3}} \right) = \overline{OL} \left( 2 \sqrt{\frac{2}{3}} \right) = \overline{OF} \left( 2 \sqrt{\frac{2}{3}} \right) \]
\[ = \overline{OB} \left( 2 \sqrt{\frac{2}{3}} \right) \]
\[ = \overline{RL} = \overline{LF} = \overline{FR} = \overline{RB} = \overline{LB} = \overline{FB} \]
Likewise
\[ \overline{AB} = \overline{FB} \sqrt{\frac{2}{3}} \]
\[ \overline{AB} \sqrt{\frac{3}{2}} = \overline{FB} = \overline{RL} = \overline{LF} = \overline{FR} = \overline{RB} = \overline{LB}. \]

**APPENDIX II. Isosceles Tetrahedron**

Consider the sagittal section of the isosceles tetrahedron shown in fig. 10. This section is obtained by passing a plane through \( B \) and \( F \) perpendicular to the frontal plane. Point \( O \), the center of the frontal plane, lies in the plane of this section. In the isosceles tetrahedron
\[ \overline{RL} = \overline{LF} = \overline{FR}, \quad \overline{OR} = \overline{OL} = \overline{OF} = \overline{OB} = r, \]
and
\[ \overline{RB} = \overline{LB} = \overline{FB} \]
Since the frontal plane is an equilateral triangle with medians \( \overline{OF}, \overline{OR}, \) and \( \overline{OL} \).
\[ \overline{OR} \sqrt{3} = \overline{OL} \sqrt{3} = \overline{OF} \sqrt{3} = \overline{OB} \sqrt{3} \]
\[ = \overline{RL} = \overline{LF} = \overline{FR}. \]

**APPENDIX III. Cube**

In the case of the cube shown in fig. 6
\[ \overline{FQ} = \overline{FR} = \overline{QS} = \overline{RS} = \overline{TU} = \overline{UW} = \overline{VW} \]
\[ = \overline{TV} = \overline{PT} = \overline{UQ} = \overline{WS} = \overline{VR} \]
and
\[ \overline{OP} = \overline{OQ} = \overline{OR} = \overline{OS} = \overline{OT} = \overline{OV} = \overline{OU} = \overline{OW} \]
But
\[ \overline{UR} = 2 \overline{OR} = 2 \overline{OQ} = \sqrt{\overline{UQ}^2 + \overline{QS}^2 + \overline{RS}^2} \]
Therefore,

$$2\overline{OR} = \overline{RS} \sqrt{3}$$

or

$$\frac{2}{\sqrt{3}} \overline{OR} = \overline{RS}, \text{ etc.}$$

REFERENCES


Standardizing Factors in Electrocardiography
JAMES A. CRONVICH, JOHN P. CONWAY and GEORGE E. BURCH

Circulation. 1950;2:111-121
doi: 10.1161/01.CIR.2.1.111

Circulation is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231
Copyright © 1950 American Heart Association, Inc. All rights reserved.
Print ISSN: 0009-7322. Online ISSN: 1524-4539

The online version of this article, along with updated information and services, is located on the World Wide Web at:
http://circ.ahajournals.org/content/2/1/111

Permissions: Requests for permissions to reproduce figures, tables, or portions of articles originally published in Circulation can be obtained via RightsLink, a service of the Copyright Clearance Center, not the Editorial Office. Once the online version of the published article for which permission is being requested is located, click Request Permissions in the middle column of the Web page under Services. Further information about this process is available in the Permissions and Rights Question and Answer document.

Reprints: Information about reprints can be found online at:
http://www.lww.com/reprints

Subscriptions: Information about subscribing to Circulation is online at:
http://circ.ahajournals.org//subscriptions/