An Evaluation of Starr’s Equation for the Prediction of Stroke Volume

By Travis Bridwell, Rachel M. Jenss, Sc.D., and David G. Greene, M.D.

Starr’s formula for deriving stroke volume from pulse pressure, diastolic pressure, and age has been tested experimentally in a group of miscellaneous hospital patients. Nearly simultaneous measurements of stroke volume by the direct Fick method and of brachial arterial pressure by direct puncture were analyzed. The measured stroke volume frequently differed considerably from the values calculated from the formula. Certain theoretic limitations of the application of multiple regression equations to this material are presented.

The determination of cardiac stroke volume by the Fick principle is a laborious procedure. If this important physiologic datum could be obtained more simply and still retain the accuracy of the Fick technic, such simplification would be of benefit to the physician. Starr, Schnabel, Askovitz, and Schild attempted to predict stroke volume with a multiple-regression-type mathematical equation. Simulating 46 systoles with known stroke volumes in 6 cadavers, they recorded simultaneous intra-arterial pressures. Employing 5 measurements, 61 equations were derived to determine which combination of measurements gave the smallest error of prediction.* These measurements included diastolic pressure, pulse pressure, pulse wave velocity, body surface, and age. The equation that gave the smallest error included pulse pressure (PP), diastolic pressure (DP), and age, as follows:

$$\text{Stroke volume} = 91.0 + 0.54 \text{PP} - 0.57 \text{DP} - 0.61 \text{age}$$

In two thirds of the estimates with this equation the error was less than 5.9 ml. Jackson has devised a nomogram for the easy application of this formula to patients.

A priori it would seem that mathematical manipulation of systoles simulated in cadavers is far removed from the determination of a dynamic quantity like cardiac stroke volume in the living subject. The present analysis was undertaken to test the validity of this procedure.

Methods and Results

Data were collected on 28 miscellaneous hospitalized patients in whom stroke volume was measured by means of the Fick principle and right-sided cardiac catheterization. Each patient was in a resting state. Two stroke volume determinations were conducted 20 min. apart. The average difference between the 2 determinations of stroke volume was 7.1 ml., or less than 12 per cent of the mean stroke volume for the whole group of 60 ml. There was a variance of the difference of 6.7 ml. For each determination duplicate intra-arterial pressures were recorded photographically with a Hathaway blood pressure recording system. Subjects ranged in age from 23 to 72 and exhibited varying disease states. Included were 11 with chest disease, of whom 6 had cor pulmonale, 5 with rheumatic heart disease, 4 with arteriosclerotic heart disease, 3 with hypertensive cardiovascular disease, 4 with normal cardiovascular systems, and 1 with thyrotoxicosis. It is in such miscellaneous patients that an easy determination of stroke volume would be most helpful.

Starr’s equation was applied to our data. Since duplicate pressure readings had been taken for each stroke-volume determination the equation was applied to each of these 104

From the Departments of Medicine, Physiology, and Preventive Medicine and Public Health, University of Buffalo School of Medicine; the Buffalo General Hospital; and the Division of Biostatistics, Erie County Health Department, Buffalo, N. Y.

This study was supported in part by a grant from the John A. Hartford Foundation.

* In these computations the equation of the best line is found by the method of least squares. The error of this method, as defined by the standard error, can be readily computed.
readings. A standard error of 24.0 ml was obtained from the line of perfect agreement when Starr's equation was applied to our data. This compares with his predicted error of 5.9 ml.

Another approach was to derive our own equation from our own data. The method of least squares presented by Mills was used. This method assumes all the error to be in the dependent variable and none in the independent variables. The equation derived from our data is:

\[ \text{Stroke volume} = 66.0 + 0.34 \, PP - 0.11 \, DP - 0.36 \, \text{age} \]

The standard error of our own equation as applied to our own data was 24.3 ml.

**DISCUSSION**

Of the many influences on stroke volume one would expect age to be one of the least important. Diastolic pressure also bears no immediately obvious relation to stroke volume. It is therefore not surprising that formulas based on these 2 variables do not predict stroke volume well. Since neither Starr's equation nor one similarly derived by us yields useful predictions of stroke volume in a representative cross section of patient population, the further pursuit of this subject does not appear profitable.

There is, however, the additional question of statistical theory. It is not so much a matter of taking data and running them through an equation as it is deciding whether or not it is correct to use a multiple regression equation. Multiple-regression analysis is a statistical tool that enables one to study the effect of multiple, independent variables upon a dependable variable. What Starr and his group have done is to take 5 factors—diastolic pressure, pulse pressure, pulse wave velocity, body surface area, and age—and derive 61 multiple-regression equations by using these factors in all possible combinations. They found that the equation with pulse pressure, diastolic pressure, and age provided the best basis for the most accurate estimation. The criterion by which they judged the worth of the various equations was the standard error of the line of best fit according to the method of least squares.

A standard error in and of itself may be misleading over a wide axis. It is conceivable that there might be ranges of stroke volumes where close agreement existed between the actual and estimated stroke volumes. In such an instance an estimating equation would still have value. To see whether this was true a graph was made to compare actual and estimated stroke volume (fig. 1). The disparity between actual and estimated values is apparent. Inspection suggests that the data come from 2 overlapping populations with 2 separate regressions. The cases have been examined with this in mind, and no correlation with other known characteristics has been discovered.

The use of multiple-regression equations is based upon several assumptions that must be satisfied for the results to be valid. An important requisite is that each independent variable be linearly related to the dependent variable. For example, in order to include age in the equation, stroke volume must either increase or decrease linearly as age increases. Just from an empiric basis it is hard to imagine that a linear relation exists between stroke volume and age.

From our data diastolic pressure, pulse pressure, and age were plotted against stroke volume (figs. 2-4). It is apparent from these figures that a linear relation does not exist.

A second fault in the equation is that the 3
STARR'S EQUATION FOR PREDICTION OF STROKE VOLUME

![Figure 2](image)

**FIG. 2.** Scattergram showing the relationship of stroke volume (SV) in ml. obtained by the direct Fick method, with the diastolic pressure in mm. Hg obtained from graphic records. Each dot represents 1 pair of observations. Note the absence of a linearity.

“independent variables” are not independent of one another. Pulse pressure is defined as the difference between systolic pressure and diastolic pressure. Here pulse pressure is distinctly dependent upon the diastolic pressure, so that the equation does not really use 3 independent variables, but 2. A mistake in the determination of the diastolic pressure will influence the pulse pressure also.

If one overlooks the absence of linearity and the dependence of pulse pressure on diastolic pressure, the material may be examined on other grounds. Comparison of Starr's equation and our own in tables 1 and 2 reveals what appear to be differences between the coefficients. One might expect closer agreement between the coefficients if Starr's fundamental idea was applicable to our data. Another interesting observation is that the same accuracy is achieved whether Starr's equation or our own equation is used on our data.

The stroke volumes in our group of subjects

![Figure 3](image)

**FIG. 3.** Scattergram showing the relationship of stroke volume (SV) in ml. obtained by the direct Fick method with the pulse pressure in mm. Hg obtained from graphic records. Each dot represents 1 pair of observations.

![Figure 4](image)

**FIG. 4.** Scattergram showing the relationship of stroke volume (SV) in ml. obtained by direct Fick method with age in years. Each dot represents 1 pair of observations. Note the absence of a linearity.

**TABLE 1.**—Formulas for Prediction of Stroke Volume

<table>
<thead>
<tr>
<th></th>
<th>SV =</th>
<th>PP</th>
<th>DP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starr and co-workers</td>
<td>91.0</td>
<td>0.54</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>Present authors</td>
<td>66.0</td>
<td>0.34</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.36</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2.**—Coefficients of Formulas for Prediction of Stroke Volume

<table>
<thead>
<tr>
<th></th>
<th>PP</th>
<th>DP</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Starr and co-workers</td>
<td>.54</td>
<td>.57</td>
<td>.61</td>
</tr>
<tr>
<td>(2) Present authors</td>
<td>.34</td>
<td>.11</td>
<td>.36</td>
</tr>
<tr>
<td>Ratio (per cent) (2) × 100</td>
<td>63</td>
<td>19</td>
<td>59</td>
</tr>
</tbody>
</table>
ranged between 21 and 115 ml., with a median value of 56 ml. In dealing with a physiologic variable of this magnitude, an error of 5.9 ml. might be quite acceptable for many purposes. But an error of 24 ml. seems so large that it would be difficult to use such variable data.

**Summary**

The application of Starr's multiple-regression equation for the prediction of stroke volume to 104 intra-arterial pressures obtained from 28 resting patients resulted in a standard error of 24.0 ml. rather than the 5.9 ml. obtained by Starr. Application of an independently derived equation to our own data resulted in a standard error of 24.3 ml.

To employ a multiple-regression equation validly several conditions must be fulfilled, 1 of which is a linear relation between each independent variable (diastolic pressure, pulse pressure, and age) and the dependent variable (stroke volume). This requisite has not been satisfied in our data. It is apparent that Starr's equation is not sufficiently accurate to yield meaningful estimates of stroke volume in a heterogeneous group of hospital patients.

**Summario in Interlingua**

Le equation Starr a regression multiple pro le prediction del volumine pulsar esseva applicate a 104 pressiones intra-arterial obtenite ab 28 patientes in stato de reposo. Le resultante error standard esseva 24,0 ml plus tosto que le 5,9 ml obtenite per Starr. Le application de un equation de derivation independente resultava in un error standard de 24,3 ml.

Le uso valide de un equation a regression multiple presuppone le satisfaction de plure conditiones. Un de istos es le existentia de un relation linear inter cata un del variables independente (pression diastolic, pression pulsar, etate) e le variable dependente (volumine pulsar). In nostre datos iste condition non esseva satisfacite. Il es clar que le equation Starr non es sufficientemente accurate pro le calculation de significative estimationes del volumine pulsar in un grupo heterogenee de patientes hospitalisate.

**References**


---


It is often difficult to determine by palpation whether a pulsating liver is due to tricuspid insufficiency, to the impact of the contracting right ventricle, or to the expanding subdiaphragmatic aorta. In the majority of instances, simple physical examination will make the correct diagnosis. The fingers of one hand are placed on a carotid artery, while the other hand presses gently upon the enlarged liver. If the pulsations are transmitted by a ventricular or aortic impact, both carotid and hepatic pulsations are felt to occur simultaneously; if due to tricuspid insufficiency, the hepatic pulsation is felt after the carotid pulsation, imparting a see-saw sensation to the examining fingers. The accuracy of this simple procedure has been checked by recording characteristic insufficiency pressure curves from the right atrium during cardiac catheterization in man and by postmortem examination.

**Kitchell**
An Evaluation of Starr's Equation for the Prediction of Stroke Volume
TRAVIS BRIDWELL, RACHEL M. JENSS and DAVID G. GREENE

Circulation. 1956;14:250-253
doi: 10.1161/01.CIR.14.2.250
Circulation is published by the American Heart Association, 7272 Greenville Avenue, Dallas, TX 75231
Copyright © 1956 American Heart Association, Inc. All rights reserved.
Print ISSN: 0009-7322. Online ISSN: 1524-4539

The online version of this article, along with updated information and services, is located on the World Wide Web at:
http://circ.ahajournals.org/content/14/2/250

Permissions: Requests for permissions to reproduce figures, tables, or portions of articles originally published in Circulation can be obtained via RightsLink, a service of the Copyright Clearance Center, not the Editorial Office. Once the online version of the published article for which permission is being requested is located, click Request Permissions in the middle column of the Web page under Services. Further information about this process is available in the Permissions and Rights Question and Answer document.

Reprints: Information about reprints can be found online at:
http://www.lww.com/reprints

Subscriptions: Information about subscribing to Circulation is online at:
http://circ.ahajournals.org/subscriptions/