General Networks for Central Terminals in Electrocardiography and Vectorcardiography

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A method is described for obtaining true orthogonal components of the heart vector by measuring the differences in potential between resistor networks which are connected to the limbs and to the back. The values of the resistors in the networks are determined by solving equations into which the scalar coefficients for the limbs and for the back are inserted.

Coefficients for defining the relationships between the manifest potential difference of the heart (the heart vector) and leads from the surface of the body (lead vectors) have been determined on models. The agreement in values obtained by two teams of investigators in two different parts of the world on two different models is quite remarkable. Before these data can be applied clinically, one must attempt, by determining the coefficients in man, to overcome some of the objections to experiments on a model.

Execution of such clinical research poses many difficulties. Preliminary direct approaches in one dimension have been made by placing a dipole within the heart or in the esophagus. Theoretic solutions have been proposed by Frank and by McFee and Johnston. Observations by Frank, based on studies of a full scale model of a man, have only the objection already mentioned. Analyses presented by McFee & Johnston are based on the relationships between the "lead field" and the lead voltage. From these relationships they propose to create specifically desired leads either by cancelling out unwanted direction, flare or curvature of the lead field by combination with other leads, or by adjustment with suitable resistor networks of the lead field obtained at the surface of the body for the purpose of producing the desired field in the heart.

From a practical point of view, the principal objection to these methods is the difficulty in visualizing the three-dimensional distribution of the lead field in the body. Furthermore, the last method is not likely to have appeal for the clinician since its application will probably involve the use of multiple electrodes, variable in number and location, and also involve the use of multiple resistors.

In an attempt to overcome these objections, and at the same time to elaborate on the analytic methods available for constructing accurate "null-potential" electrodes, it will be shown that uniplanar central terminals ("component terminals") used in conjunction with a biplanar null-potential terminal may be used to determine the orthogonal components (x, y, z) of the spatial heart vector in man. The central terminal principle is used for several reasons. One is that vectorcardiographic reference systems which utilize it are less sensitive than those that do not to inter- and intra-individual changes in the position of the dipole.

Using the general theory of heart vector projection, a mathematical solution for two-dimensional and three-dimensional central terminals may be devised. The absolute values of the resistors within the networks of the terminals are not unique but the ratios of the values are, and are determined from the trans-

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fer functions* of the medium (the body). Points on the medium must be chosen with certain restrictions depending on the type of terminal desired (see below). The transfer functions of the three components can be calculated from the networks, the image space heart vector synthesized, and the true spatial vectorcardiogram created. The remaining problem of determining the transfer functions of the heart vector in the human body will be detailed elsewhere; the results can be no more accurate than the accuracy of these determinations.

It is not the purpose of this presentation to restate the general theory of heart vector projection begun by Burger and van Milaan and ably elaborated by Frank. The objective here is to apply the theory to the design of general networks, to obtain accurate central terminals and to show how these may be used to obtain the heart vector free of the errors inherent in calculations made from conventional surface leads.

Definition of Terms

Using the notations of Frank, the equivalent heart dipole will be represented by the vector \( \vec{p} \) where

\[
\vec{p} = \hat{i}p_x + \hat{j}p_y + \hat{k}p_z
\]

(1)

and \( \hat{i}, \hat{j} \) and \( \hat{k} \) are the three component unit vectors.

The heart vector, \( \vec{p} \), and its three scalar components \( p_x, p_y \) and \( p_z \) are functions of time. The midpotential of \( \vec{p} \) is assigned the value of zero. All potentials are measured from this point and it is the potential of this point that must be duplicated at the zero potential central terminal. The potential difference, \( V \), between it and any other point in or on the surface of the medium is given by

\[
V = \vec{c} \cdot \vec{p} = c_xp_x + c_y p_y + c_z p_z
\]

(2)

The lead vector, \( \vec{c} \), is invariant with time and is only dependent upon the boundary condi-

\* A "transfer function" may be described as a set of coefficients by means of which input and output quantities of any network may be related. The term is synonymous with the scalar coefficients defined in models by Burger and van Milaan and by Frank. 

The Central Terminal in Two Dimensions

It is unimportant to define the shape or character of the medium for which the central terminal is being determined, inasmuch as the vector, \( \vec{c} \), in itself contains all pertinent data. It can properly be called a transfer function for the problem specified. It is only necessary that enough of these functions be involved and the leads which determine them chosen so that the transfer functions are basically independent as stated in the theory of orthogonal functions. Roughly, in two dimensions, three points not on a straight line are needed. To be truly at zero potential the points must define a bounded plane which passes through the center of the dipole. On the other hand, if the terminal is to be used to obtain one orthogonal component of the heart vector, then it is only necessary to select the points so that the bounded plane defined by them is penetrated by the axis of the component to be defined.

Figure 1 illustrates a general two-dimensional inhomogeneous medium where three points,

![Figure 1](image-url)
1, 2 and 3, specified completely by their transfer functions, \( \delta_1, \delta_2 \) and \( \delta_3 \), will be used to determine the terminal that will have the same potential as the midpoint of \( \bar{p} \). A schematic representation of the terminal circuit is illustrated in figure 2. \( V_1, V_2 \) and \( V_3 \) are the three scalar potentials obtained at the points 1, 2 and 3. They are measured from these points to the midpoint of \( \bar{p} \). The circuit is made up of three resistors, \( R_1, R_2 \) and \( R_3 \), having respectively currents \( i_1, i_2 \) and \( i_3 \) flowing through them. The central terminal, noted as \( M \), must have the same potential as the center point of \( \bar{p} \) at all times and for all possible variations of \( \bar{p} \) if it is to be used as a true null potential electrode. Since \( M \) must have the same potential as the midpoint of \( \bar{p} \) it follows that

\[
\begin{align*}
V_1 &= i_1 R_1 \\
V_2 &= i_2 R_2 \\
V_3 &= i_3 R_3
\end{align*}
\]

and also from Kirchoff’s law that:

\[
i_1 + i_2 + i_3 = 0
\]

Solving equations (3) and (4) will uniquely determine the resistor network for obtaining the central terminal. For (3) to be true for all values of \( \bar{p} \), it has to hold for each of the components separately. Substituting (3) into (4) initially for a unit vector for \( \bar{p} \) in the \( x \) direction, then for a unit vector in the \( y \) direction, two simultaneous equations are obtained.

\[
\begin{align*}
c_{1x} + c_{2x} + c_{3x} &= 0 \\
c_{1y} + c_{2y} + c_{3y} &= 0
\end{align*}
\]

Since there are two equations with three unknowns, the absolute values of the resistors are not determined but their ratios to one another are. Solving for these ratios.

\[
\begin{align*}
R_1 &= \frac{c_{1x} c_{2y} - c_{2x} c_{1y}}{c_{2x} c_{3y} - c_{3x} c_{2y}} \\
R_2 &= \frac{c_{1x} c_{2y} - c_{2x} c_{1y}}{c_{3x} c_{1y} - c_{1x} c_{3y}}
\end{align*}
\]

An example of this network can be worked out using the results for a male torso as given by Frank.9

\[
\begin{align*}
V_1 &= V_R = -51 p_x - 57 p_y + 27 p_z \\
V_2 &= V_L = 25 p_x - 84 p_y + 41 p_z \\
V_3 &= V_F = -21 p_x + 91 p_y + 11 p_z
\end{align*}
\]

In this example only three points are utilized for a three-dimensional heart vector. The \( z \) component of the vector is neglected and a central terminal obtained for the two-dimensional case involving only the \( x \) and \( y \) components. Substituting into (6) and (7)

\[
\begin{align*}
R_1 &= \frac{(-51)(-84) - (25)(-57)}{(25)(91) - (-21)(-84)} = 5700 \\
R_3 &= \frac{(-51)(-84) - (25)(-57)}{(-21)(-57) - (-51)(91)} = 5838
\end{align*}
\]

This indicates that \( R_2 \) should be made approximately equal to \( R_3 \) and that \( R_1 \) should be made approximately 11 times the size of \( R_3 \). In passing, it should be noted that these figures are in contrast to those of Bayley and Schmidt.11 No agreement would be expected if the electric boundaries of the body, upon which the magnitude of the transfer functions in part depends, were changed by the immersion experiments done by these investigators, even though the immersion fluid was tap water.

As noted, the \( z \) component has not been taken into consideration and the central terminal obtained above will have a \( z \) component.
with respect to the true midpotential of the heart vector. Only the \( x \) and \( y \) components will be reduced to zero. This point will be of considerable interest when vectorcardiographic reference systems are discussed.

The Central Terminal in Three Dimensions

In three dimensions it is necessary to specify four points with four transfer functions. This is illustrated in figure 3 where four points

\[ \text{Fig. 3. Four points are shown (1, 2, 3 and 4) specified by their transfer functions } (\bar{c}_1, \bar{c}_2, \bar{c}_3 \text{ and } \bar{c}_4) \text{ surrounding the heart vector } \bar{p} \text{ situated in a heterogeneous three-dimensional medium.} \]

(1, 2, 3 and 4) specified by their transfer functions, \( \bar{c}_1, \bar{c}_2, \bar{c}_3 \text{ and } \bar{c}_4 \), surround the heart vector, \( \bar{p} \).

Figure 4 shows schematically the resistor network that is used to determine the central terminal, \( M \), which has the same potential as the midpoint of \( \bar{p} \). It now entails the use of four resistors. Again \( V_1, V_2, V_3 \text{ and } V_4 \) are the four scalar potentials obtained at the points 1, 2, 3 and 4. These potentials are measured from these points to the midpoint on the heart vector. The network is now made up of four resistors \( R_1, R_2, R_3 \text{ and } R_4 \) having the currents \( i_1, i_2, i_3 \text{ and } i_4 \) flowing through them respectively. Since \( M \)

\[
\begin{align*}
V_1 &= i_1 R_1 \\
V_2 &= i_2 R_2 \\
V_3 &= i_3 R_3 \\
V_4 &= i_4 R_4
\end{align*}
\]

(9)

Since there is no sink or source at \( M \),

\[
\begin{align*}
i_1 + i_2 + i_3 + i_4 &= 0
\end{align*}
\]

(10)

![Fig. 3. Four points are shown (1, 2, 3 and 4) specified by their transfer functions \( \bar{c}_1, \bar{c}_2, \bar{c}_3 \text{ and } \bar{c}_4 \) surrounding the heart vector \( \bar{p} \) situated in a heterogeneous three-dimensional medium.](image)

![Fig. 4. A schematic representation of the three-dimensional central terminal network. \( M \) is the central terminal in a resistor network connected to four points in or on the surface of a heterogeneous medium.](image)

As in the two-dimensional case, in order that equation (9) be true for all values of \( \bar{p} \), it has to hold for each of the components separately. Substituting (9) into (10) first for a unit vector for \( \bar{p} \) in the \( x \) direction, then for a unit vector for \( \bar{p} \) in the \( y \) direction, and finally for a unit vector for \( \bar{p} \) in the \( z \) direction, three simultaneous equations are obtained:

\[
\begin{align*}
\frac{c_{1x}}{R_1} + \frac{c_{2x}}{R_2} + \frac{c_{3x}}{R_3} + \frac{c_{4x}}{R_4} &= 0 \\
\frac{c_{1y}}{R_1} + \frac{c_{2y}}{R_2} + \frac{c_{3y}}{R_3} + \frac{c_{4y}}{R_4} &= 0 \\
\frac{c_{1z}}{R_1} + \frac{c_{2z}}{R_2} + \frac{c_{3z}}{R_3} + \frac{c_{4z}}{R_4} &= 0
\end{align*}
\]

(11)
Since there are three equations and four resistors the ratios of the resistors to one another can be determined. Solving (11)

\[
\begin{align*}
\frac{R_1}{R_4} & = \frac{c_{1x} c_{2y} c_{3z} + c_{2x} c_{3y} c_{1z} + c_{3x} c_{1y} c_{2z}}{c_{2x} c_{2y} c_{4z} + c_{3x} c_{1y} c_{4z} - c_{2x} c_{3y} c_{4z} - c_{3x} c_{2y} c_{4z}} \\
\frac{R_2}{R_4} & = \frac{c_{1x} c_{2y} c_{3z} + c_{2x} c_{3y} c_{1z} + c_{3x} c_{1y} c_{2z}}{c_{2x} c_{4y} c_{1z} + c_{3x} c_{4y} c_{1z} - c_{2x} c_{3y} c_{1z} - c_{3x} c_{2y} c_{1z}} \\
\frac{R_3}{R_4} & = \frac{c_{1x} c_{2y} c_{3z} + c_{2x} c_{3y} c_{1z} + c_{3x} c_{1y} c_{2z}}{c_{1x} c_{2y} c_{4z} + c_{1x} c_{4y} c_{2z} + c_{3x} c_{2y} c_{4z} - c_{2x} c_{3y} c_{4z} - c_{3x} c_{1y} c_{4z}}
\end{align*}
\]

and

\[
\begin{align*}
R_1 & = 315,954 \\
R_4 & = -135,238 = -2.336 \\
R_2 & = 315,954 \\
R_4 & = -475,998 = -0.664 \\
R_3 & = 315,954 \\
R_4 & = -478,956 = -0.660
\end{align*}
\]

This can only mean that the four points chosen on the model do not include the center of the dipole. Frank’s three-dimensional drawing of the image space does not show that the dipole (center of the image space) lies anterior to the plane made by the first three points (R, L, F).

Vectorcardiographic Reference Systems

In vectorcardiography it is necessary to obtain the three components of the heart voltage in as pure a form as possible. When the transfer functions are known, it has been shown that the components of the heart vector can be obtained from the scalar voltages measured at the surface of the heterogeneous medium by means of simultaneous equations. This is awkward and makes the problem of obtaining the components in “real time” a difficult one. “Real time” means the instant at which they occur in the body. The central terminal in three dimensions gives an electric terminal which is at zero potential for all components, provided the image space, delimited by the four points selected in the medium, surrounds the source of potential.

A terminal remains to be obtained at which only one of the heart vector components exists. This has already been accomplished. In the example of the two-dimensional central terminal it was pointed out that the z component was completely neglected. This means that the voltage in the z axis will not be zero at this two-dimensional central terminal and the difference in potential between this terminal and the true central terminal should consist of only the z component. The actual amplitude of the component may be calculated very simply once the network has been established.

General Vectorcardiographic Network

In the general case, in addition to the three-dimensional central terminal shown in figure 4, three other two-dimensional central terminals have to be determined. These are illustrated in figure 5. A superscript is employed at the central terminal and on the resistors involved which designates the component which is to be obtained from that network. Three terminals are employed for each of the two-dimensional circuits. They can be con-
connected to different combinations of points, as shown in the figure, or they may be connected to the same points; or they may have various connections, depending upon how large a component is available in each case. The networks are not interchangeable and have to be recalculated for each set of points used.

$M^z$ is the “component” terminal for $z$. The resistors $R_1^z$, $R_2^z$, and $R_3^z$ have been chosen in the ratios specified by the two-dimensional central terminal equation (8), so that the $x$ and $y$ components are zero at $M^z$. This means that between $M^z$ and the central terminal, $M$, only the $z$ component exists. It should be noted again that the three points for a two-dimensional terminal must define a plane through which the axis of the component to be determined must pass. By calculating the difference in potential that exists between $M^z$ and $M$ for a unit voltage, $P_z$, substituted for the heart vector, it is possible to obtain the transfer function for the $z$ component that appears at $M^z$. Since this voltage only has one component, the transfer function can be designated by a scalar, $c^z$. Similarly the transfer function for the $x$ component voltage which appears between $M^x$ and $M$, designated as $c^x$, and for the $y$ component voltage which appears between $M^y$ and $M$, designated as $c^y$, can be obtained.

Voltage between $M^x$ and $M = V_x = c^x P_x$
Voltage between $M^y$ and $M = V_y = c^y P_y$ (16)
Voltage between $M^z$ and $M = V_z = c^z P_z$

Solving for the heart vector components,

$$ p_x = \frac{V_x}{c^x} \quad p_y = \frac{V_y}{c^y} \quad p_z = \frac{V_z}{c^z} \quad (17) $$

The values of the transfer functions of the component terminals may be readily taken into account in a practical application by means of fixed amplification or attenuation of the component terminal voltages. The resultant heart vector can then be used to generate the true, spatial vectorcardiogram.

**Summary**

Using the general theory of heart vector projection, a general solution for both two-dimensional and three-dimensional central terminals is given. Examples of the networks, using Frank’s coefficients determined from models, are given.

In the case of the two-dimensional terminal

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**Fig. 5.** For vectorcardiography, in addition to the three-dimensional central terminal shown in figure 4, three other two-dimensional (or component) terminals must be employed (see text).
only two of the three \((x, y, z)\) components of the heart vector can be balanced out. The third component will affect that terminal then referred to as a "component terminal." Between the component terminal and the three-dimensional central terminal voltages proportional to the unbalanced component of the former can be obtained. By calculating the transfer functions for the respective component terminals it is possible to obtain the true components of the heart vector for vectorcardiography.

As better methods for determining the transfer functions or scalar coefficients for leads from points on the surface of the human body are developed, the resultant voltages obtained from the networks suggested should closely approach those of the true heart vector within the limits of the assumptions made in the general theory of projection of this vector.

**SUMMARIO IN INTERLINGUA**

Es describite un metodo pro obtener ver componentes orthogon del vector cardiac per mesurar le differentias de potential inter retes resistential que es connectite con le extremidades e con le dorso. Le valores del resistentias in le retes es determinate per solver equationes in le quales le coefficientes scalar pro le extremidades e pro le dorso ha esse inserite.

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