A Direct Experimental Study of Three Systems of Spatial Vectorcardiography

By Ernest Frank, Ph.D.

Three commonly used systems of vectorcardiography are compared on an absolute and direct quantitative basis in terms of equations, geometric interpretations, scalar leads and vectorgrams, using experimental results obtained on an accurate, three-dimensional homogeneous torso model of the human subject with a dipole fixed in position in the center of the heart. It is concluded that the accuracy of the three systems, as presently used, is unsatisfactory and that further consideration of the systems of Duchosal and Grishman is questionable because of excessive intrinsic errors. Certain chance properties of the Wilson tetrahedron enable standardization modifications which lead to fairly accurate results for some dipole positions, and this system also possesses other features which appear to be desirable. The applicability of the results and conclusions to the human subject depends upon the degree to which the heart might be represented by a fixed-position dipole and the degree to which human body electrical inhomogeneities influence body surface potentials.

RECOGNITION that the electrical forces in the heart are three-dimensional in nature has precipitated an international, active quest for accurate means of recording their spatial aspects. A variety of electrode arrangements on the body surface of the human subject are presently being utilized by many electrocardiographers.1-6 Most of these systems of vectorcardiography are founded upon the assumptions that the human torso is a substantially homogeneous volume conductor and that the heart forces may be represented by a single fixed-position, time-varying current dipole. The degree to which these assumptions are applicable to the human subject has not been firmly established. Additional assumptions are usually made by the advocates of various specific systems concerning the geometry of the body and the position of the dipole. However, it has been shown in this laboratory by exhaustive studies on human torso models that the shape of the human torso and the position of the equivalent heart dipole are important factors which introduce large errors in currently employed methods of vectorcardiography, even granting that the torso medium is homogeneous and that the heart forces may be consolidated into a single fixed-position dipole.

It is the purpose of this paper to extract from the tremendous amount of data obtained with male and female homogeneous torso models, moulded accurately to the human subject, representative findings which pertain to a few of the commonly used systems of spatial vectorcardiography. It is hoped that the shortcomings of all of these systems will be recognized by workers in this field so that they may be encouraged to follow more fruitful lines of investigation.

SYSTEMS OF VECTORCARDIOGRAPHY

If the assumption of a fixed-position dipole representation of the human heart is adopted, this is tantamount to assuming that there is essentially only a single entity to be determined on an individual human subject; namely, the three-dimensional variations of the dipole moment. This single entity then represents the total and complete information which is sought from body surface measurements. *

* Precordial electrodes might give additional information if the dipole approximation becomes
THREE SYSTEMS OF SPATIAL VECTORCARDIOGRAPHY

Fig. 1. A rectangular coordinate system defining three planes commonly used in vectorcardiography is illustrated. The nomenclature for these planes, and for the directions and letter designations of the rectangular axes has not been standardized officially.

The spatial dipole variations can be decomposed into three parts, for convenience in recording and study, by considering the projections on three rectangular axes, shown in figure 1, from which the spatial variations may be synthesized by considering various projections of the component variations. From a mathematical point of view, the single entity may be symbolized by the vector \( \mathbf{p} \),* the dipole moment which is a function of time. This heart dipole can be expressed in terms of its rectangular components as \( \mathbf{p} = i p_x + j p_y + k p_z \), where \( p_x \), \( p_y \), and \( p_z \) are the components of the dipole and \( i, j \) and \( k \) are standard unit vectors of rectangular coordinates.8 Thus, the problem of vectorcardiography, granting the dipole assumption, is to determine \( p_x \), \( p_y \), and \( p_z \) simultaneously as functions of time.

significantly inaccurate when electrodes are located close to the heart. The magnitude of this "proximity effect" has not yet been firmly established, but it is probably small.7

* Throughout this paper, vectors are indicated by boldface type.

This has been the objective of various electrocardiographers doing research in this field.

Any three independent potential differences measured on the body surface can, in principle, lead to a determination of \( p_x \), \( p_y \) and \( p_z \), and the results obtained from any desired electrode arrangements should all agree,9 provided the data are properly analyzed on a sound basis. From this standpoint it may be seen that the choice of electrode placement is really a practical one including such factors as reproducibility of electrode position, influence of dipole position at the electrode sites selected, and influence of body build. The results obtained for the three systems to be described, each of which employs different electrode positions, are not in agreement because of the inapplicability of the assumptions which are made concerning the relationship between the electrode potentials and the heart dipole; that is, the effects of dipole position and torso shape are not taken into account properly.

**Duchosal-Sulzer System.** The electrode arrangement advocated by Duchosal and Sulzer is illustrated in figure 2. The details regarding exact placement of electrodes \( n \), \( o \), \( p \), \( q \) may be found elsewhere.1 In this system it is assumed that the potential difference between \( p \) and \( o \) is proportional only to the \( x \) component of the dipole, \( p_x \); that the potential difference between \( o \) and \( n \) is proportional only to the \( y \) component, \( p_y \); and that the potential difference between \( o \) and \( q \) is proportional only to the \( z \) component, \( p_z \). Moreover, it is assumed that the constant of proportionality is the same for each electrode pair. These ideas may be expressed compactly in terms of the following three equations, using the \( xyz \) coordinate system of figure 1.

\[
\begin{align*}
V_p - V_o &= cp_x, \\
V_o - V_n &= cp_y, \\
V_o - V_q &= cp_z
\end{align*}
\]

(1)

where \( V_n \), \( V_o \), \( V_p \) and \( V_q \) are the potentials of the electrodes with respect to the dipole midpotential and \( c \) is a constant of proportionality.

**Grishman-Scherlis System.** A modification of the Duchosal-Sulzer electrode arrangement has been advocated by Grishman and Scherlis, as illustrated in figure 2, but the assumptions
are retained concerning the equal proportionality between the potential differences and the dipole component to which the line joining the physical electrodes is parallel. A description of the exact electrode placement may be found elsewhere. The equations assumed to be applicable to this system are

\[ V_2 - V_1 = kp_x, \quad V_1 - V_4 = kp_y, \quad V_1 - V_3 = kp_z, \]  

\[ V_2 - V_1 = kp_x, \quad V_1 - V_4 = kp_y, \quad V_1 - V_3 = kp_z, \]  

where \( V_1, V_2, V_3 \) and \( V_4 \) are the electrode potentials with respect to the dipole mid-potential, and \( k \) is a constant of proportionality, assumed to be the same for each electrode pair.

The treatment of the potential differences in this manner in both the Duchosal and Grishman systems is based upon such considerations as the directions of the anatomic lines joining the electrodes, the anatomic angles subtended by the electrodes from the heart center, and the anatomic distances from the electrodes to the heart center. However, it will be shown that this basis is not accurate for these electrode arrangements. Moreover, it has been shown elsewhere that anatomic distances and angles are generally very distorted as far as electrical effects are concerned for practically all points on the surface of the torso.

**Wilson Tetrahedron System.** The electrode arrangement advocated by Wilson and associates is shown in figure 2. Three electrodes R, L, and F are the standard limb electrodes and electrode B is located on the back approximately 1.0 inch to the left of the seventh dorsal vertebra. The treatment of the potential differences in this system is based rigidly upon a homogeneous conducting sphere with a centric dipole, which is about as arbitrary

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**Fig. 2.** Schematic illustration of the electrode arrangements employed in three commonly used systems of vectorcardiography. Letter and number identifications for the electrodes as originally presented are retained. The solid squares indicate electrodes placed on the back of the subject. The standard limb electrodes are indicated by R, L and F. A central terminal C is connected through equal resistors, r, to R, L and F.
as the anatomic basis discussed above. Two types of theoretical tetrahedrons inscribed in the sphere have been advocated, the equilateral tetrahedron and the isosceles tetrahedron, even though the electrode placement on the human subject is the same in both cases. In the equilateral tetrahedron the dipole at the center of the sphere is equidistant from each of the four points R, L, F and B on the surface of the sphere; the dipole is at the center of the tetrahedron and the frontal-plane triangle is located in front of the dipole parallel to the frontal plane. In the isosceles tetrahedron the centric dipole lies in the plane of the frontal-plane triangle and again the distances from R, L, F and B to the dipole are equal. The mathematical equations obtained depend upon which theoretic model is used; the equilateral tetrahedron will be discussed here since it gives the best agreement in scalar-lead wave shape with results in torso models. For the equilateral tetrahedron it has been shown\(^{3}\) that the following unipolar voltages are produced by a centric dipole in the homogeneous conducting sphere:

\[
\begin{align*}
V_R &= K(\sqrt[3]{6} p_x - \sqrt{2} p_y - p_z) \\
V_L &= K(\sqrt[3]{6} p_x - \sqrt{2} p_y - p_z) \\
V_F &= K(0 p_x + 2\sqrt{2} p_y - p_z) \\
V_B &= K(0 p_x + 0 p_y + 3p_z)
\end{align*}
\]  

where \(V_R, V_L, V_F\) and \(V_B\) are the potentials of the electrodes with respect to the dipole mid-potential, arbitrarily assigned the value zero, and \(K\) is a constant of proportionality.

A study of these four equations indicates that potential differences proportional to only one component of the dipole may be obtained theoretically by the use of a central terminal connected from R, L and F through three equal resistors, \(r\), as shown in figure 2.* This corresponds to the “Equilateral Tetrahedron, Case II,” analyzed by Cronvich.\(^{11}\) It is interesting to note that the potential of this central terminal is not theoretically zero, being given by

\[
V_C = \frac{1}{3}(V_R + V_L + V_F) = -Kp_z
\]  

assuming the resistors, \(r\), are very large in comparison with the resistance behind the electrodes. It may be seen from equations 3 and 4 that the following potential differences are obtained for theoretically determining the three dipole components:

\[
\begin{align*}
V_L - V_R &= 2K\sqrt[3]{6} p_x \\
V_F - V_C &= 2K\sqrt{2} p_y \\
V_B - V_C &= 4Kp_z
\end{align*}
\]  

In contradiction to the other systems discussed, it will be noted that these potential differences do not have equal proportionality constants.

Treatment of the potential differences in this manner, based on a theoretical model of a centric dipole in a homogeneous conducting sphere, is considerably erroneous\(^{12}\) since the human torso is not spherical in shape and the heart dipole is not centrically located. It will be shown that the tetrahedron which applies to a torso with an eccentric dipole departs drastically from that inscribed in the sphere.

**EXPERIMENTAL METHOD**

By the use of a homogeneous conducting torso with an immersed fixed-position dipole, it has been possible to obtain detailed information enabling the dipole variations \(p_x, p_y\) and \(p_z\) to be calculated precisely from any designated electrode positions on the torso surface. It has been found convenient to summarize the results in the form of an image surface\(^{9,10}\) which, in addition to enabling any system of vectorcardiography to be analyzed, gives insight into many aspects of other problems faced in electrocardiography. With these data it can be shown that exactly the same dipole variations are deduced regardless of the electrode positions used. However, if the assumptions made by the advocates of various systems are applied to their electrode positions, results are obtained which do not agree with those which were determined by direct experiment, and, furthermore, each system entails errors of different kind and degree. These errors are superimposed upon those which are already inherent in the assumptions concerning the dipole representation of the heart and the homogeneity of the conducting medium.

The experimental apparatus and method is described in detail elsewhere.\(^{12-15}\) For this particular study, the finite dipole was fixed in position within the homogeneous conducting torso in the center of the region occupied in life by the ventricular mass during very deep inspiration. True unipolar voltages were determined in both male and female torso
models at approximately 200 boundary electrode positions on each. From these data, which have been analyzed comprehensively and expressed in the form of image surfaces, the voltages pertaining to the 12 electrode positions shown in figure 2 were singled out for presentation here for a male torso of average build (chest: 40 inches, hips: 38 inches, waist: 32½ inches). Photographs of this torso model are shown in figure 3. The relationship between the unipolar voltages at the points n, o, p, q, R, L, F, B and the dipole immersed within the torso could be determined directly since the dipole was located in a known and accurately-prescribed manner. The data obtained are accurate to approximately 5 per cent.

RESULTS

The complete results may be summarized succinctly in the form of equations which relate the true unipolar voltage of the electrode in question to the three components of the heart dipole. These equations are given in table 1, where the numerical coefficients are in the same relative units. The equations in table 1 contain the complete results of this study. While they possess the advantages of completeness and brevity, it is recognized that their implications may not be fully appreciated without a detailed discussion. Therefore, they will be interpreted in a variety of ways in the section which follows.

INTERPRETATION OF RESULTS

Bipolar Equations. The bipolar equations which pertain to each of the three systems may be obtained by subtracting appropriate unipolar voltages. The central terminal voltage may be computed from table 1 for the torso model as follows:

\[ V_c = \frac{1}{3} (V_R + V_L + V_F) \]

\[ = -16p_x - 17p_y + 26p_z \]
Carrying out the necessary subtractions, the bipolar results given in table 2 are obtained from table 1. A comparison of the equations in table 2 with those claimed to apply for the various systems, as shown in equations 1, 2 and 5, reveal glaring discrepancies which become apparent at once. None of the bipolar voltages is proportional to only one component of the heart dipole, although in all cases the single components which are assumed in equations 1, 2 and 5 do have coefficients which are larger than the disturbing (undesired) coefficients. The bipolar equations which pertain to the Duchosal and Grishman systems have far from equal proportionality factors c and k as assumed in equations 1 and 2, and the relative proportionality factors theoretically called for in equation 5 for the Wilson system do not agree with those for the torso model. A quantitative measure of the proportionality factors is given later in figure 6 in which the relative amplitude factors required for each bipolar lead to give a best fit for an assigned dipole variation has been noted. The central terminal voltage for the torso model, given in equation 6, can also be seen to be far from agreement with the theoretical result of equation 4.

An approximate measure of the faithfulness of recording the correct shape of a given dipole component for each lead (regardless of the relative amplitude) may be obtained by comparing the disturbing coefficients with the coefficient of the dipole component which is presumed to be measured in a given lead. For example, Vp - Vs is presumably proportional to p2 only according to equation 1 but, from table 2, it can be seen that the coefficients of p2 and p2 are 7 and 5, respectively, which correspond to 33 per cent and 24 per cent of the coefficient of p2, which is 21. Following this line of reasoning, the results shown in table 3 are obtained. These per cent disturbing coefficients do not constitute an exact measure of the shape distortion of a given lead, since p2, p2 and p2 are each different functions of time. The disturbing terms may tend to cancel or reinforce one another, depending upon the instantaneous magnitude and polarity of p2, p2 and p2. However, these percentages are usually indicative of the shape errors. For instance, in figure 6, which shows specific typical functions for p2, p2 and p2, the reasonably good shape agreement for all three systems in the case of p2 could be expected from the fact that the per cent disturbing coefficients are all relatively small (5 to 14 per cent). Also, in the case of p2, the good agreement in shape for the Wilson and Duchosal systems and the relatively poor agreement in shape for the Grishman system is indicated by the per cent disturbing coefficients of table 3. In the case of p2, an example is provided which shows that the signs of the disturbing co-

### Table 1.—Unipolar Equations

<table>
<thead>
<tr>
<th>Duchosal</th>
<th>Grishman</th>
<th>Wilson</th>
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<tbody>
<tr>
<td>V0 = -30p2 - 65p2 + 34p2</td>
<td>V1 = -50p2 + 62p2 + 24p2</td>
<td>VR = -51p2 - 57p2 + 27p2</td>
</tr>
<tr>
<td>Vp = -32p2 + 83p2 + 14p2</td>
<td>V2 = 16p2 + 82p2 + 46p2</td>
<td>Vp = -21p2 + 91p2 + 11p2</td>
</tr>
<tr>
<td>Vs = -11p2 + 90p2 + 19p2</td>
<td>V3 = -51p2 + 75p2 - 7p2</td>
<td>VB = -16p2 - 13p2 + 90p2</td>
</tr>
<tr>
<td>Vs = -34p2 + 85p2 + 3p2</td>
<td>V4 = -44p2 - 60p2 + 35p2</td>
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### Table 2.—Bipolar Equations

<table>
<thead>
<tr>
<th>Duchosal</th>
<th>Grishman</th>
<th>Wilson</th>
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<tbody>
<tr>
<td>Vp - Vs = 21p2 - 7p2 + 5p2</td>
<td>V2 - V1 = 66p2 + 20p2 + 22p2</td>
<td>Vp - Vr = 76p2 - 27p2 + 14p2</td>
</tr>
<tr>
<td>Vp - Vs = 7p2 + 148p2 - 20p2</td>
<td>V1 - V4 = -6p2 + 122p2 - 11p2</td>
<td>Vp - Vc = -5p2 + 108p2 - 15p2</td>
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<tr>
<td>Vp - Vs = 2p2 - 2p2 + 11p2</td>
<td>V1 - V3 = p2 - 13p2 + 31p2</td>
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### Table 3.—Per Cent Disturbing Coefficients

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<thead>
<tr>
<th>Heart-Dipole Component Desired</th>
<th>Duchosal</th>
<th>Grishman</th>
<th>Wilson</th>
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Fig. 4. Geometric representation of the electrode positions of Duchosal and Grishman in image space for a homogeneous torso with an immersed dipole. The heart dipole is located at the origin of the coordinate system. The potential difference between any pair of electrodes is obtained by projecting the heart dipole onto the vector joining the corresponding points in image space, and multiplying by the length of this vector. It may be seen in both qualitative and quantitative terms that these electrodes depart considerably from their anatomic positions.

Efficients must also be considered, for the Wilson system yields a faithful $p_v$ shape despite the existence of sizable disturbing coefficients. The disturbing terms tend to cancel almost completely in this case for the dipole functions illustrated in figure 6.

Geometric Interpretation. It has been shown elsewhere$^9$ that the unipolar equations given in table 1 may be expressed as the vector dot product of the heart dipole vector $p$ with a vector $c$ whose components are equal to the coefficients. For example, the unipolar voltage at electrode $n$ may be written as

$$V_n = c_n \cdot p = (-39i - 65j + 34k) \cdot (ip_x + jp_y + kp_z)$$  (7)

The purpose of expressing the equations in this form is to enable a geometric interpretation. $V_n$ may be looked upon as arising from the projection of $p$ onto $c_n$, multiplied by the length of $c_n$. The constant vectors $c$ for each of the unipolar voltages in table 1 may be constructed from a common origin and the tips of the vectors are the image points which correspond to the physical points on the torso. The 12 electrode image points for the systems under study here are shown in figures 4 and 5, where vector projection is valid, and are seen to depart drastically from the points in physical space on the torso surface. This means that the use of vector projection in physical space leads to large errors. The vectors joining the points in image space can be used to obtain the bipolar voltages by projecting $p$ onto them and multiplying by their length. The basis for this vector projection scheme has been discussed in detail elsewhere.$^9$

In terms of the geometric representation in figure 4, the defects of the Duchosal and Grishman systems may be seen in graphic terms. The
advocates of these systems claim in effect that the vectors in image space joining \( o, p \) and \( 1, 2 \) are parallel to the \( x \) axis; that the vectors joining \( o, n \) and \( 1, 4 \) are parallel to the \( y \) axis; and that the vectors joining \( o, q \) and \( 1, 3 \) are parallel to the \( z \) axis. That is, the points \( o, p \) and \( 1, 2 \) are presumed to coincide in the sagittal view; the points \( o, n \) and \( 1, 4 \) are presumed to coincide in the transverse view; and the points \( o, q \) and \( 1, 3 \) are presumed to coincide in the frontal view. Moreover, it is assumed that the lengths of all of these vectors are equal. In addition, it is assumed that the dipole, which is located at the origin of the coordinate system of figure 4, is equidistant from the four points of each system. It is clearly evident that these properties are not applicable to a homogeneous torso with a dipole in the heart center. The reader may assess for himself the magnitude of the errors implied by the diagram of figure 4 which has quantitative significance.

The "equilateral" tetrahedron of Wilson is shown as it appears in image space in figure 5. It is clear that the tetrahedron is not equilateral. Moreover, the heart dipole is located considerably in front of the limb-lead triangle rather than at the center of the tetrahedron as supposed in the spherical model. Further, the vector joining \( R \) and \( L \) is not parallel to the \( x \) axis (\( R \) and \( L \) do not coincide in the sagittal view), the vectors from the heart dipole (point \( O \) to \( F \) is not parallel to the \( y \) axis and the vector from \( O \) to \( B \) is not parallel to the \( z \) axis.

By a fortuitous combination of effects, however, the Wilson system is not subject to errors as great as the Duchosal and Grishman systems, despite the glaring discrepancy between the tetrahedron in image space and the one inscribed in the idealized sphere. It will be noted in figure 5 that the tip of the central-terminal vector, drawn from the dipole (at the origin) to the median point of the limb-lead triangle, is nearly coincident with \( B \) in the frontal view, indicating that the vector between \( B \) and \( C \) is very nearly parallel to the \( z \)-axis and, therefore, that the potential difference \( V_u - V_r \) will be very nearly proportional to \( p_u \). This has also been indicated in table 3. Furthermore, the location of \( C \) with respect to point \( F \) in the transverse view shows that the vector joining \( C \) and \( F \) is more nearly parallel to the \( y \) axis than the vector through \( F \) and \( O \), the true zero reference potential. Thus, the imperfection of the central terminal again tends to improve matters. In addition, while the vector from \( R \) to \( L \) is far from parallel to the \( x \) axis, it is so oriented in space (for this particular dipole position) that the contributions of the \( y \) and \( z \) components of the dipole tend to oppose one another. This may also be seen from the equation \( V_u - V_r \) in table 2. The \( p_r \) leads of the other two systems do not possess this lucky property. As a result of these chance condi-

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* The image surface of a homogeneous conducting sphere with a centric dipole is undistorted as compared with the physical sphere.\(^n\)
tions, the tetrahedron, though departing drastically from the theoretical assumptions of Wilson, has certain potentialities which will be discussed in detail later.

**QRS Complex Illustration.** As a final method of interpreting the results of the torso-model studies, the scalar lead voltages and vectorcardiograms produced by a specific three-dimensional dipole variation may be compared, as they would be recorded by each system.\(^1\)

For this purpose the dipole is assigned variations which are believed to be applicable for a normal subject during the QRS complex, although the results are not critically dependent upon the particular choice of dipole variations. The same considerations apply for the P and T waves. The dipole functions \(p_x\), \(p_y\) and \(p_z\) which have been selected for illustrative purposes are shown by the solid lines in figure 6 and the three projections of the dipole loops are given by the solid lines of figure 7.

(a) Scalar lead comparison. A comparison between the calculated bipolar lead voltages produced on the torso by the variable-moment, fixed-position dipole and the corresponding dipole component is given in figure 6. The relative amplitudes in each case have been adjusted for the best fit; the vertical arrows indicate the points at which the scalar leads of the various systems were forced to agree with the instantaneous amplitude of the dipole component. In order to accomplish this normalization, the standardization factors shown on figure 6 were used. These factors give a measure of the amplitude error involved in each system when the advocated standardization factors are employed. It may be seen that all three systems give fair agreement in shape with \(p_x\), but that only the Wilson system gives acceptable agreement in shape in the case of \(p_z\). In the case of \(p_z\), Grishman's system can be seen to be significantly inferior to either of the other two systems. The cause for the disagreement in shape can be seen from tables 2 and 3 where the scalar leads produced by each system are seen to be molested by dipole components other than those presumed to be reflected in a particular lead.

(b) Vectorcardiograms. A more exacting comparison of the fidelity of each system may be made in terms of the vectorcardiograms shown in figure 7 which would be recorded by using the standardization factors called for by the advocates of each system rather than those which are deliberately selected to make amplitudes agree based upon the torso behavior, as was done in figure 6. These vectorcardiograms are those which are obtained on the torso model applying equations 1, 2 and 5; that is, \(V_p - V_o\), \(V_o - V_n\), \(V_o - V_q\) are recorded at equal amplification; \(V_2 - V_1\), \(V_1 - V_4\), \(V_1 - V_3\) are recorded at equal amplification; \(V_p - V_c\) is recorded at \(\sqrt{3}/2\) times the amplification of \(V_L - V_n\), and \(V_h - V_c\) is recorded at \(\sqrt{3}/2\).
times the amplification of $V_L - V_R$. In all cases the relative amplitudes of each system have been adjusted to agree with $p_s$; i.e., if the amplification used for $V_L - V_R$ is taken as unity, then the amplification for the Duchosal $p_x$ lead, $V_p - V_o$, is 1.97 and that of the Grishman $p_x$ lead, $V_3 - V_1$, is 0.60.

It can be seen that the Duchosal system tends to elongate the loop enormously in the head-to-foot direction, and to contract drastically the chest-to-back component. The system of Grishman gives relatively close agreement (3 per cent) in the relative amplitudes of the $p_x$ and $p_y$ components, but the shape distortion of $p_x$, shown in figure 6, causes the frontal-plane loop to depart from the heart-dipole loop shape. The Grishman system also gives a contracted view of the chest-to-back component of the dipole. The Wilson system, while giving fair to good agreement in all of the scalar-lead wave shapes for this dipole position, does not reproduce the dipole vector loops faithfully, largely because the standardization factors called for by the spherical model are not suitable. The standardization factor for $V_p - V_c$ is too large by a factor of about $\sqrt{3}/0.75 = 2.3$, and that for $V_b - V_c$ is also too large by a factor of about $\sqrt{3}/2/0.98 = 1.25$. The effects of reducing the standardization factors are presented in the discussion.

**Discussion**

A detailed description of the results obtained with a homogeneous torso model of the human subject with a dipole immersed in the center of the heart has been given and compared with results that are obtained with three commonly used systems of vectorcardiography. None of these systems, when treated in accordance with the assumptions of their advocates, gives a faithful record of the true dipole variations. This conclusion is based upon direct experimental measurements. The same conclusion applies for a female torso, and for a wide variety of positions of the dipole within the heart volume. Thus, granting the assumptions of a fixed-dipole representation of the human heart and of a homogeneous conducting torso, it is concluded that these systems as presently used are not satisfactory.

Not all of the systems yield the same distortion, however. From the standpoint of reproduction of scalar-lead shape only, disregarding the relative amplitudes of each lead, they may be rated in the following order of descending merit: (1) Wilson, (2) Duchosal, (3) Grishman. An approximate quantitative evaluation can be made from table 3 by means of the percentage disturbing coefficients for each scalar lead, or a specific illustration may be found in figure 6. The relative amplitudes of these systems are also in error, but, by the use of empirical standardization factors, these may be corrected for, to a partial extent. In view of this comparison which applies for a homogeneous torso with a dipole in the center of the heart, it would seem clear that the systems of Duchosal and Grishman do not warrant further study, since it is extremely unlikely that nondipolar properties of the human heart generator and inhomogeneities of the body could conceivably offset the intrinsic defects shown in the torso model. Moreover, it should be remembered that Duchosal and Grishman, along with most vectorcardiographers, postulate a dipole for the heart and assume a homogeneous medium.

On the other hand, the relatively good agreement in scalar-lead shape for all three leads of the Wilson system, resulting from a fortuitous set of conditions, suggests that the scale factors experimentally determined for the torso might be applied to the Wilson system with substantial improvement. The scale factors which are more nearly appropriate, based on the model results for this particular dipole position, are given in figure 8 with the corresponding modified vector loops. Specifically, $V_L - V_R$ and $V_b - V_c$ are recorded at equal amplification while $V_p - V_c$ is recorded at $3/4$ the amplification of the other two. The other two systems cannot be improved to this extent by modifying the scale factors, and it is very important to appreciate just why the modified Wilson system gives results that are as satisfactory, as shown in figure 8. It is not because the theory of a centric dipole in a homogeneous conducting sphere is accurately applicable to the human torso, since, as has been shown, the tetrahedron in image space departs drastically from that inscribed in the theoretical model of Wilson. The good agreement is traceable to two factors: First, the Wilson central terminal volt-

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* The standardization factors suitable for a female torso with a dipole in the center of the heart differ by 10 per cent or less from those given for the male torso.
age deviates from the dipole midpotential in such a way as to tend to offset the errors that would otherwise be obtained, and second, the orientation of the vector from R to L in image space tends to give error cancellation to a fair degree for this particular dipole position. The effectiveness of these lucky error-cancellation tendencies depends to some extent on the shapes of \( p_x, p_y \) and \( p_z \) and more markedly upon the dipole position, as has been emphasized elsewhere.\(^{15,16} \) Generally, the potential difference between R and L is more susceptible to errors of dipole position, torso shape and dipole functions than are the potential differences between F and C, and B and C.

An illustration of the dependence on dipole position is given in figure 9 for the frontal-plane vector loop, using the same \( p_x, p_y \) and \( p_z \) as in figure 8. The angle at which the manifest heart vector is a maximum is, in this case, 65 degrees as compared with the true angle of 44 degrees at which the heart dipole magnitude, as seen in the frontal plane, is a maximum. This discrepancy is considerable despite the use of the same standardization factors which gives good agreement for a dipole in the center of the heart.

Viewing the image surface as a whole, it is possible to determine torso electrode positions which are substantially perfect for recording faithfully the variations of the separate dipole components; to a degree which is superior to that of the modified Wilson tetrahedron.\(^{10,17} \) However, when the dipole position or torso shape is changed, these same electrode positions are no longer as accurate. Any system of fixed electrode placement for all subjects will be vulnerable to errors traceable to torso shape variations and dipole positional changes. However, it can be seen from the results presented here that the central terminal can be used to advantage in such a way as to offset the effects of dipole position to some extent. In fact, an extremely important practical principle can be formulated; namely, systems which do not employ a central terminal, or do not pool the electrodes together in some fashion, are more highly vulnerable to dipole positional changes from subject to subject, and within the same subject, than those which take advantage of the averaging effects of a central terminal of.

Fig. 8. Vectorcardiograms obtained in a homogeneous torso with a dipole fixed in position in the center of the heart, using modified standardization factors for the Wilson tetrahedron, are compared with projections of the dipole variations. The relative agreement between recorded variations and actual dipole variations is approximately 20 per cent or better for this dipole position.

Fig. 9. The influence of dipole position is illustrated for a frontal-plane vectorcardiogram using the modified Wilson system of figure 8. The dipole position here was shifted from that in figure 8 in the following manner: 2.4 cm. to the left \((x \text{ direction})\), 2.4 cm. toward the foot \((y \text{ direction})\) and 1.5 cm. toward the anterior chest \((-z \text{ direction})\). The largest error is in \( V_L - V_R \) which is more susceptible to dipole positional changes than \( V_F - V_C \); the former error depends upon the partial cancellation of \( p_x \) and \( p_y \) in the equation for \( V_L - V_R \), while the latter error depends upon the degree to which the Wilson central terminal potential shifts as a result of the dipole positional change. Any system of vectorcardiography using the same body-electrode positions regardless of heart position is vulnerable to errors of the type illustrated.
some kind. This principle has nothing to do with whether the central terminal is at the di-
pole midpotential or not; indeed, the compen-
sating effects of the central terminal come
about for the very reason that the junction is
not at the dipole midpotential. Since the Wil-
ton tetrahedron incorporates this important
feature and, moreover, since it utilizes stand-
ard electrode positions which are easily deter-
mined and reproduced, and for which a huge
amount of correctable empirical data already
exist, it would appear to show future promise
as a practical system of electrode arrangement
for vectorcardiography, and deserves further
study.

The applicability of these results to the hu-
man subject is, of course, a crucial question,
since in these studies only the effects of dipole
position and torso shape were considered. This
question is currently being explored on human
subjects in this laboratory in general terms for
all body electrode positions, not confined to
any particular system of electrode placement.10
The applicability to the human subject of the
results presented here may be investigated by
recording body-surface voltages using all three
systems and analyzing the results using the
equations in table 2. If the torso results are ac-
curately applicable to the human subject, the
results so analyzed for each of the three systems
should all agree. Lack of agreement may not
be conclusive because the subject may not
have the same dipole position as that used to
obtain table 2 (torso data for other dipole posi-
tions may, of course, be determined experi-
mentally), and the subject’s torso shape may
differ from that shown in figure 3. On the other
hand, if these factors are properly taken into
account, lack of agreement could then be at-
tributed to errors in the fixed-dipole assump-
tion, to human body inhomogeneities, or to
faulty experimental technique which is entirely
possible in view of the many variables which
disturb the measurements such as body-loading
by the recording system, respiratory changes,
finite electrode size, and other similar factors.
Conclusions based upon the torso-model re-
results presented here must be used cautiously
until further information concerning the addi-
tional assumptions is obtained.

However, there is strong evidence provided
by unpublished data that the male torso model
results are very nearly applicable to the normal
human subject on whom the model was
moulded. It has been demonstrated, using a
generalization of the technic of Schmitt,1 that
QRS body-surface potentials on this subject
(and on several other normals as well) contain
less than 5 per cent nondipolar effects; that is,
QRS body-surface waveforms are predictable
to 95 per cent accuracy or better by means
of the dipole concept. This statement applies
to both precordial and distant electrodes.
Moreover, the amplitude and shape of mea-
sured instantaneous QRS complexes at a wide
variety of electrode sites over the entire torso
of this normal male agree to within approxi-
mately ±15 per cent with potentials predicted
from his torso model coefficients, thus indicat-
ing that the influence of human-body inho-
more.

Despite these complications, the study of
this simplified model of the human electrical
system which takes into account at least some
of the important factors governing the rela-
tionship between the human heart and the body
surface potentials it produces, has served to
show that certain systems of vectorcardiogra-
phy are significantly inferior to others. In addi-
tion it has been possible to establish a quantita-
tive estimate of the smallest errors that might
be expected in human vectorcardiography.

**Summary**

1. Three commonly used systems of vector-
cardiography are studied using a homogeneous
male torso with a dipole fixed in position in the
center of the heart.

2. A direct quantitative comparison of each
system with the known dipole behavior is made
in terms of equations, geometric interpreta-
tions, scalar leads and vectorcardiograms.

3. It is found that the scalar lead shapes of
the Wilson tetrahedron deviate, on the aver-
age, by approximately 15 per cent from the
torso dipole variations, but the scalar lead
shapes of the systems of Duchosal and Grish-
man show significantly larger discrepancies.

4. It is found that the standardization fac-
tors presently employed in the Wilson system
are too large, particularly with respect to the
head-to-foot dipole component (by a factor of
2.3). Certain standardization factors of the
other two systems are considerably greater in
error.

5. It is concluded that the accuracy of the
three systems as presently used is unsatisfactory, and that the systems of Duchosal and Grishman show least promise.

6. Certain fortuitous features of the Wilson system enable a modification of the standardization factors which leads to results that are fairly satisfactory for a dipole located in the center of the heart. This system, which possesses certain other advantages, would appear to deserve further study.

7. The applicability of these conclusions to the human subject depends upon the degree to which the behavior of dipole potentials in a homogeneous torso may be applied to humans. Methods of ascertaining this applicability are indicated.

SUMARIO ESPAÑOL

1. Tres sistemas comúnmente usados en vectocardiografía han sido estudiados usando un torso homogéneo masculino con un dipolo fijo en posición en el centro del corazón.

2. Una comparación cuantitativa directa de cada sistema con la conducta conocida del dipolo se hace en términos de ecuaciones, interpretación geométrica, derivaciones escalares y vectocardiogramas.

3. Se encuentra que las configuraciones de las derivaciones escalares del tetraedro de Wilson desvían como promedio por aproximadamente 15 por ciento de las variaciones del dipolo del torso, pero las configuraciones de las derivaciones escalares de los sistemas de Duchosal y Grishman muestran discrepancias relativamente mayores.

4. Se encuentra que los factores de normalización presentemente empleados en el sistema de Wilson son muy grandes, particularmente con respecto al componente de dipolo céfalo-caudal (por un factor de 2.3). Algunos factores de normalización de los otros dos sistemas son considerablemente mayores en error.

5. Se concluye que la precisión de los tres sistemas como usados en el presente no es satisfactoria, y que los sistemas de Duchosal y Grishman muestran la menor promesa.

6. Algunos rasgos fortuitos del sistema de Wilson proveen una modificación de los factores de normalización que conducen a resultados que son bastante satisfactorios para un dipolo localizado en el centro del corazón. Este sistema que posee algunas otras ventajas aparenta merecer más estudio.

7. La aplicabilidad de estas conclusiones al sujeto humano depende en el grado en que la conducta de potenciales dipolo en un torso homogéneo pueda ser aplicado a los humanos. Métodos para determinar esta aplicabilidad son indicados.

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A Direct Experimental Study of Three Systems of Spatial Vectorcardiography

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